A Relation between the Weyl Group $W(E_8)$ and Eight-line Arrangements on a Real Projective Plane

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Abstract

The Weyl group $W(E_8)$ of type E_8 acts on the configuration space of labelled eight lines with some conditions on a real projective plane. This configuration space is identified with an affine open subset S of $\mathbf{R}^8$. Let $\mathcal{P}_8$ be the totality of connected components of S. Then $W(E_8)$ also action on $\mathcal{P}_8$. On the other hand, to each labelled eight lines, there associates a diagram consisting of ten circles (roots in a root system of type E_8) analogous to Dynkin diagram. We already showed an existence of a $W(E_8)$-equivariant map f of the totality of such diagrams to $\mathcal{P}_8$.

The purpose of this talk is to report that the map f is injective. The first step to prove this statement is to determine all the representatives of S_8-orbits of the totality of such diagrams by using symbolic computation. There are 2160 number of S_8-orbits of the totality of such diagrams. Let $\mathcal{U}_i = w_i\mathcal{U}_1$ $(i = 1, \cdots, 2160,\ w_i \in W(E_8))$ be the representatives of S_8-orbits. The second step is to study the $W(E_8)$-actions of $\mathcal{U}_1$ to all the representatives, where $\mathcal{U}_1$ is the S_8-orbit of the remarkable diagram described in our previous paper. The third step is to determine the labelled eight lines of $f(\mathcal{U}_i)$ by operating w_i on $f(\mathcal{U}_1)$ successively. As a result, we conclude that labelled eight line contained in $f(\mathcal{U}_i)(i \neq 1)$ is not equivalent to that contained in $f(\mathcal{U}_1)$ and the injectivity of f is proved.