A Relation between the Weyl Group $W(E_8)$ and Eight-line Arrangements on a Real Projective Plane

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Abstract

The Weyl group $W(E_8)$ of type $E_8$ acts on the configuration space of labelled eight lines with some conditions on a real projective plane. This configuration space is identified with an affine open subset $S$ of $\mathbb{R}^8$. Let $\mathcal{P}_S$ be the totality of connected components of $S$. Then $W(E_8)$ also action on $\mathcal{P}_S$. On the other hand, to each labelled eight lines, there associates a diagram consisting of ten circles (roots in a root system of type $E_8$) analogous to Dynkin diagram. We already showed an existence of a $W(E_8)$-equivariant map $f$ of the totality of such diagrams to $\mathcal{P}_S$.

The purpose of this talk is to report that the map $f$ is injective. The first step to prove this statement is to determine all the representatives of $S_8$-orbits of the totality of such diagrams by using symbolic computation. There are 2160 number of $S_8$-orbits of the totality of such diagrams. Let $\mathcal{U}_i = w_i \mathcal{U}_1$ ($i = 1, \ldots, 2160$, $w_i \in W(E_8)$) be the representatives of $S_8$-orbits. The second step is to study the $W(E_8)$-actions of $\mathcal{U}_i$ to all the representatives, where $\mathcal{U}_1$ is the $S_8$-orbit of the remarkable diagram described in our previous paper. The third step is to determine the labelled eight lines of $f(\mathcal{U}_i)$ by operating $w_i$ on $f(\mathcal{U}_1)$ successively. As a result, we conclude that labelled eight line contained in $f(\mathcal{U}_i)(i \neq 1)$ is not equivalent to that contained in $f(\mathcal{U}_1)$ and the injectivity of $f$ is proved.