Affine Bracket Algebra for Automated Theorem Proving in Affine Geometry

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Abstract

Affine Bracket Algebra (ABA) is an algebraic system equipped with three basic ope rators: brackets representing areas of triangles, boundaries distinguishing between affine objects and objects at infinity, and meets representing intersections of geometric entities. ABA is defined by two classes of Grassmann-Cayley syzygies, and two classies of van der Waerden syzygies for Gröbner bases-based normalization.

The first application of ABA to automated theorem proving was carried out by the first author and his former Ph. D. student Y. Wu in 2001. Compared with the nice short proofs for projective geometr ic theorems generated by their algorithms, the proofs for affine geometric theorems are significantly long er. This puzzled the authors of this paper for years until recently when some new techniques of factorization and transformation are proposed.

In this talk, some recent new advances in automated theorem proving in affine geo metry with ABA are to be reported. A set of new geometric constructions for affine conics are established with ABA, an effective method for extracting common bracket or boundary factors is established, together with s ome transformation techniques for simplification. The algorithms are realized in Maple 10, and tested by dozens of theorems, showing significantly better performance in reducing the number of terms in algebraic com putation.