

# Algorithmic Search for Flexibility using Resultants of Polynomial Systems

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## Abstract

This talk describes the recent convergence of four topics: polynomial systems, flexibility of three dimensional objects, computational chemistry, and computer algebra.

We discuss a way to solve systems of polynomial equations with *resultants* by refining the Dixon method [KSY], [BEM] with new techniques.

Using ideas of Bricard [B] and Coutsiias [C1], [C2], we find a system of six polynomial equations in six variables and eleven parameters that models a configuration of quadrilaterals that is equivalent to some three dimensional structures. These structures are of interest in computational chemistry, as they represent molecules. We then describe and demonstrate an algorithm that examines the resultant and determines ways that the structure can be *flexible*. We review some ideas about flexibility, from Cauchy [Ca] to Bricard [B] to Connelly [Co].

Crucial in this work has been Lewis's computer algebra system "Fermat" [Lew] written especially for polynomial and matrix computations. Somewhat simliar problems were solved before in [LB] and [LS].

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