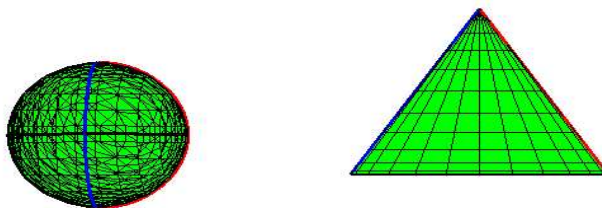


A realization of the real projective plane onto a triangle

Grosio Stanilov and Slavka Slavova
Pedagogical College Dobrich
at University of Shumen
Bulgaria

Abstract

Following F. Apery and A. Gray we investigate the antipodal map $(x_1, x_2, x_3) \rightarrow (y_1 = x_1^2, y_2 = x_2^2, y_3 = x_3^2)$ in the 3-dimensional Euclidean space. It maps the central unit half-sphere (by the standard parametrization $(\cos v \cos u, \cos v \sin u, \sin v)$, $u \in (0, \pi)$, $v \in (-\pi/4, \pi/4)$) onto a triangle with vertices $A(1, 0, 0), B(0, 1, 0), C(0, 0, 1)$. The situation is shown by the represented figures.



Every point of the triangle is a 4-fold point under this mapping and is injective in the first octant. We give an algorithm for transforming any maximal circle of the sphere onto the triangle: the images are closed curves like but not ellipses, the parametric lines are segments. This mapping gives possibility to construct a realization of the spherical (elliptical) geometry onto the triangle.