

Computing Radicals of Submodules of Free Modules over Polynomial Rings

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Abstract

After the generalization of the notion of prime ideal from the module theory, radicals of submodules in modules has been studied for the last 15 years. See [1, 2, 3].

In this study, we consider free module $M = 3DR^m$ where R is the polynomial ring of n -variables. We assume that a generating set is given for the submodule N of M and try to find a basis for radical of N . As mention in [3], even though a characterization of elements of the radical of a submodule of a free R -module is given in [2], no method for finding a generating set of radical of a submodule appears in literature.

We use the following approach to this problem. Let S be the larger polynomial rings obtained by introducing additional variables corresponding to the standard basis vectors in M . Define a map $\varphi : M \rightarrow S$. We compute radical of $\varphi(N)$ in S by known methods in polynomial rings. Then the preimage of the linear part of the radical of $\varphi(N)$ gives us the radical of N .

References

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