

# Computing Radicals of Submodules of Free Modules over Polynomial Rings

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## Abstract

After the generalization of the notion of prime ideal from the module theory, radicals of submodules in modules has been studied for the last 15 years. See [1, 2, 3].

In this study, we consider free module  $M = 3DR^m$  where  $R$  is the polynomial ring of  $n$ -variables. We assume that a generating set is given for the submodule  $N$  of  $M$  and try to find a basis for radical of  $N$ . As mention in [3], even though a characterization of elements of the radical of a submodule of a free  $R$ -module is given in [2], no method for finding a generating set of radical of a submodule appears in literature.

We use the following approach to this problem. Let  $S$  be the larger polynomial rings obtained by introducing additional variables corresponding to the standard basis vectors in  $M$ . Define a map  $\varphi : M \rightarrow S$ . We compute radical of  $\varphi(N)$  in  $S$  by known methods in polynomial rings. Then the preimage of the linear part of the radical of  $\varphi(N)$  gives us the radical of  $N$ .

## References

- [1] J. Jenkins and P.F. Smith, On the prime radicals of a module over a commutative ring, *Comm. Algebra*, **20**(12) (1992), 3593-3602.
- [2] D.P. Yilmaz and P.F. Smith, Radicals of Submodules of Free Modules, *Comm. Algebra*, **27**(5) (1999), 2253-2266.
- [3] M.E. Moore and S.J. Smith, Prime and Radical Submodules of Modules over Commutative Rings, *Comm. Algebra*, **30**(10) (2002), 5037-5067.