EXIT Charts

JASS05 –Course 4:

“The Turbo Principle in Communications”

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Overview

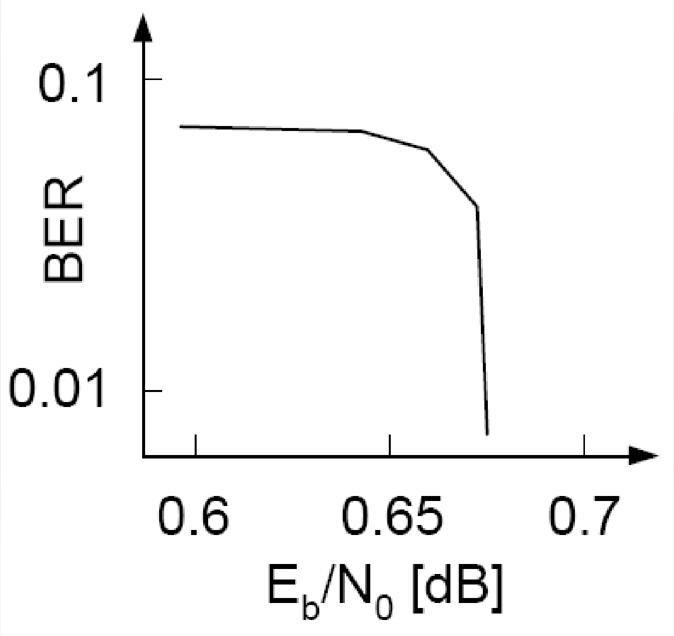
- Why EXIT Charts?
- How EXIT Charts work?
- How to draw EXIT Charts?
- Where to use EXIT Charts?
- Summary and Outlook

Why Extrinsic Information Transfer Chart?

The problem with Bit Error Rate Chart when iteratively decoding

Bad performance

- **Low** E_b/N_0 **region**
- **Turbo cliff region**

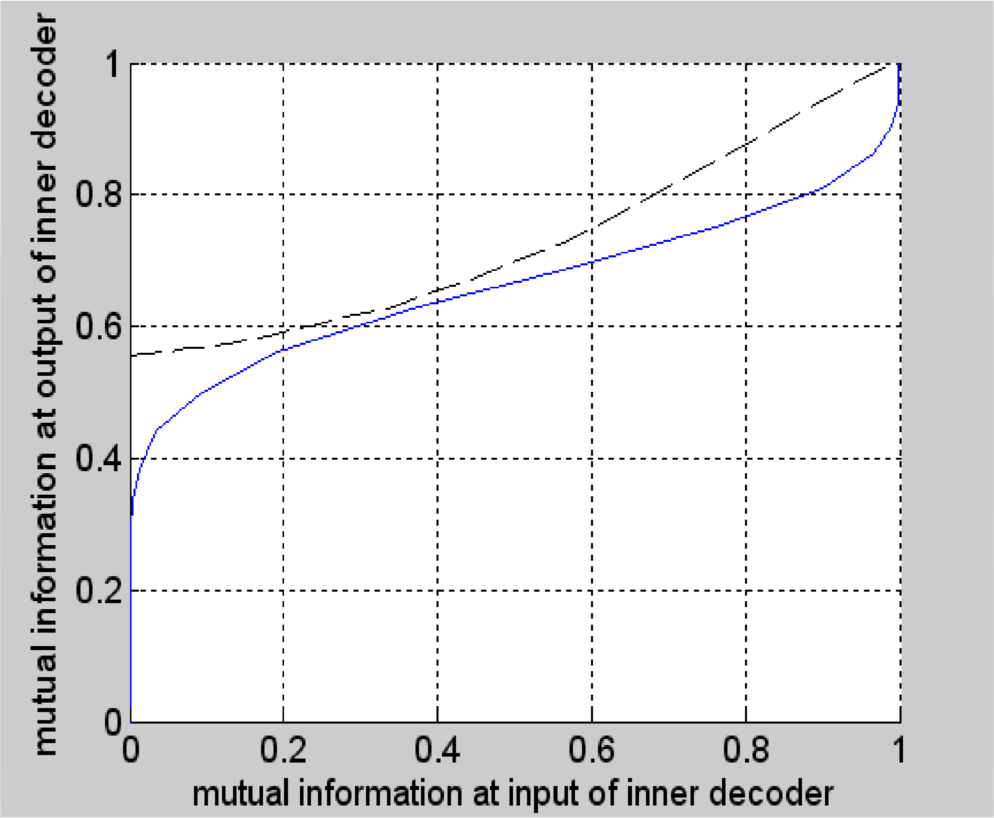


Good performance

- **BER floor region**

(Stephen ten Brink, "Convergence Behavior of Iteratively Decoded Parallel Concatenated Codes", IEEE Trans. Comm.Oct.2001)

How EXIT Charts look like?



SNR=3dB, outer code rate 0.5 [13,15]

SNR=3dB, inner code rate 0.5 [7,5]

The invention of EXIT Charts

- The exchange of extrinsic information is visualized as a decoding trajectory in EXIT Charts

- A powerful tool to visualize the convergence behavior of iterative decoding algorithms

- Good performance in low E_b/N_0 & turbo cliff region

- It provides guidance on designing good codes

How to draw EXIT Charts?

Ingredients we need:

- Mutual information
- Mutual information Transfer characteristics of iterative decoders
- Combination of transfer characteristics

Mutual information and Channel capacity

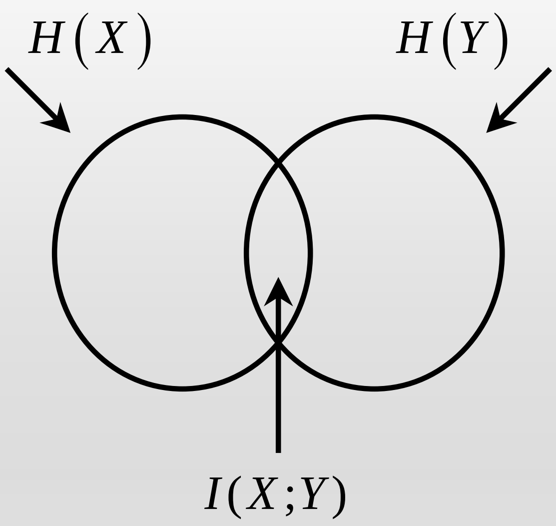
- **Mutual information**

$$I(X;Y)=H(Y)-H(Y|X)$$

$$I(X;Y)=\iint f(x,y)\log\frac{f(x,y)}{f(x)f(y)}dxdy$$

where

$$H(Y|X)=\iint f(x,y)\log\frac{1}{f(y|x)}dxdy$$



- **Channel Capacity**

$$C=\max_{p_x} I(X;Y)$$

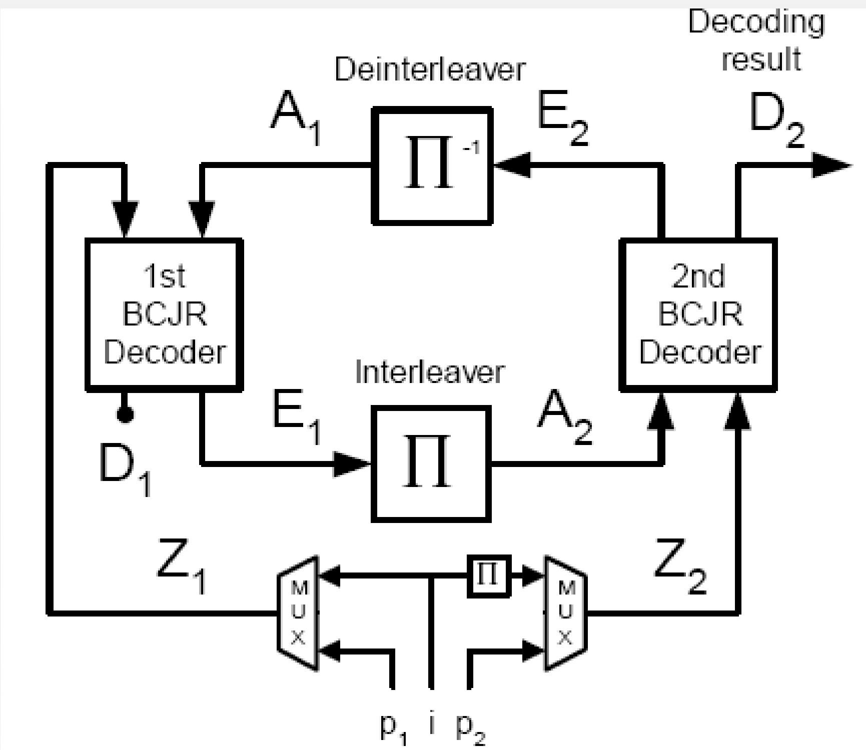
(Claude E. Shannon, "A Mathematical Theory of Communications"1948)

Why Mutual information as the parameter?

Benefits of mutual information in EXIT Charts:

- **The information-theoretic interpretation**
- **The value range and logarithmic scaling**
- **Robustness of the shape**
- **An interesting fact is the area property**

Iterative decoder used in parallel Concatenated Codes



$$E_1(D_1 - Z_1 - A_1) \rightarrow A_2$$

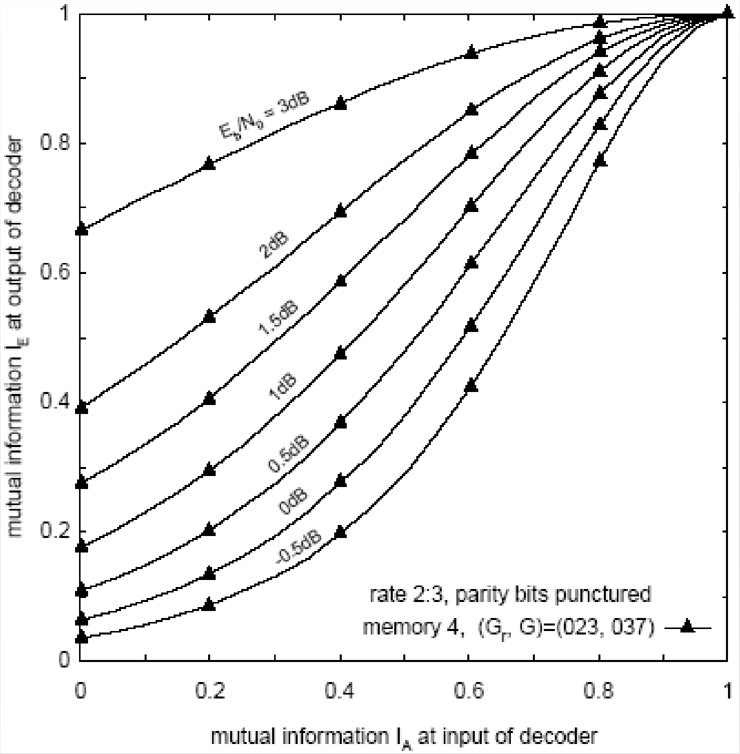
$$E_2(D_2 - Z_2 - A_2) \rightarrow A_1$$

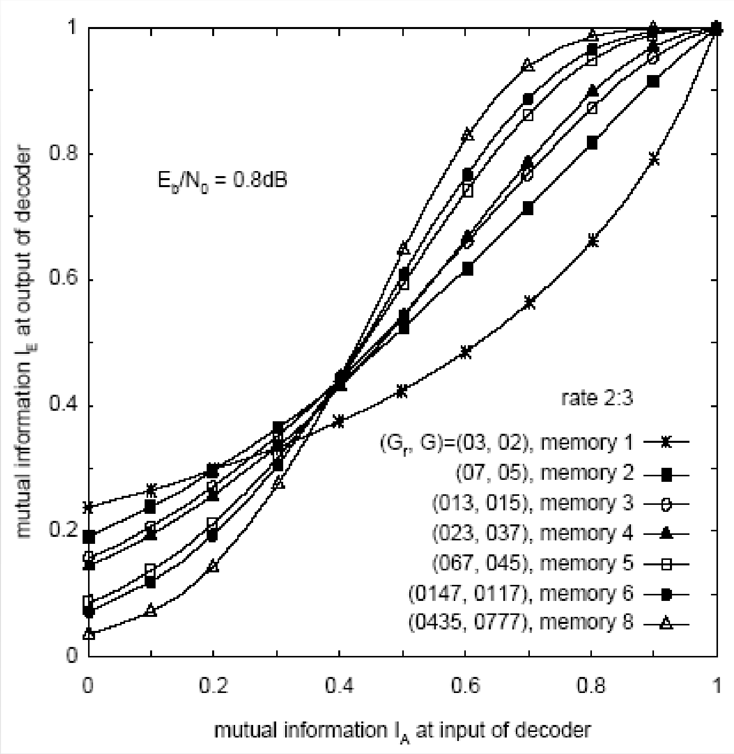
(Stephen ten Brink, "Convergence Behavior of Iteratively Decoded Parallel Concatenated Codes", IEEE Trans. Comm.Oct.2001)

Transfer characteristics

- **Iterative decoders exchange message between extrinsic and a priori** $E_1 \rightarrow A_2, \quad E_2 \rightarrow A_1$
- **Tracking of messages depends only on mutual information** $I\left(L_E; X\right), \quad I\left(L_A; X\right)$
- **Some parameters, influencing the transfer characteristic curves (number of code memory, depth of interleaving, different code polynomials and** E_b/N_0 **)**

Transfer characteristics (cont.)





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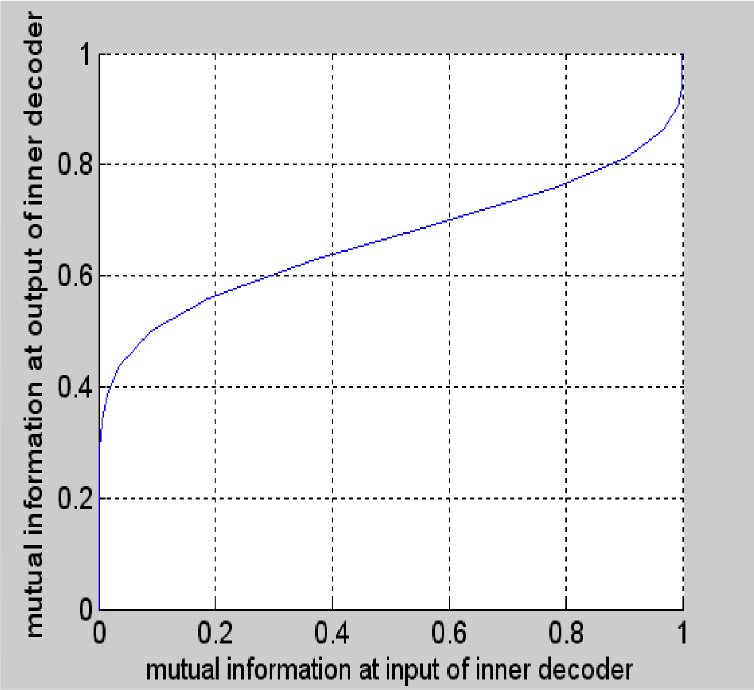
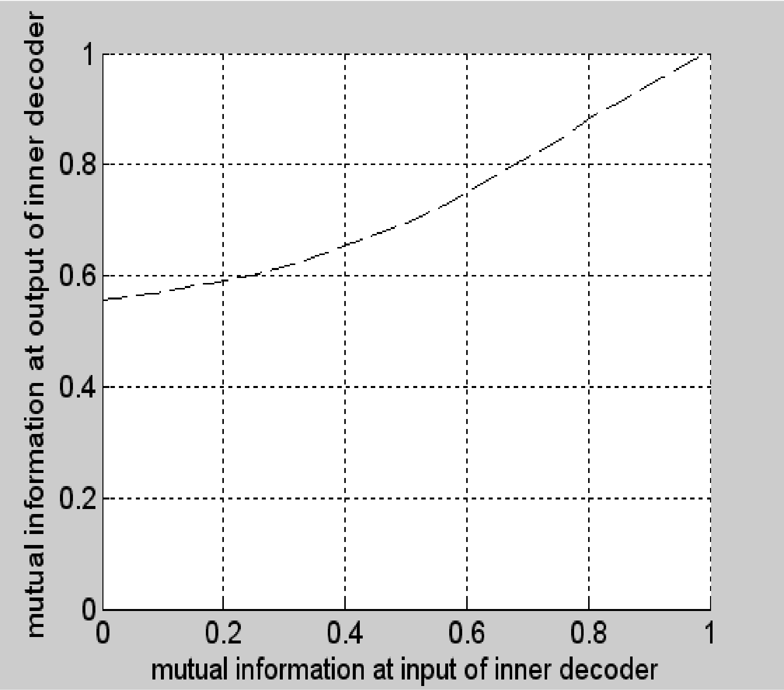
A Simple Example on Transfer Characteristics

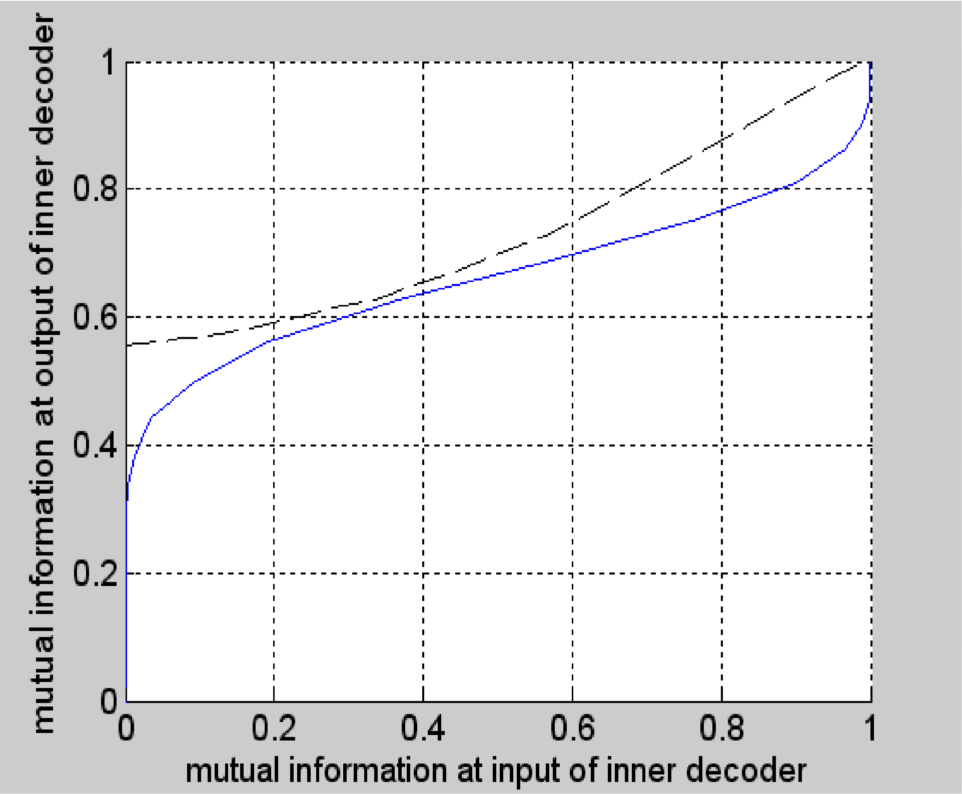




SNR=3dB, outer code rate 0.5, [13,15]

SNR=3dB, inner code rate 0.5, [7,5]

Combination of transfer characteristics

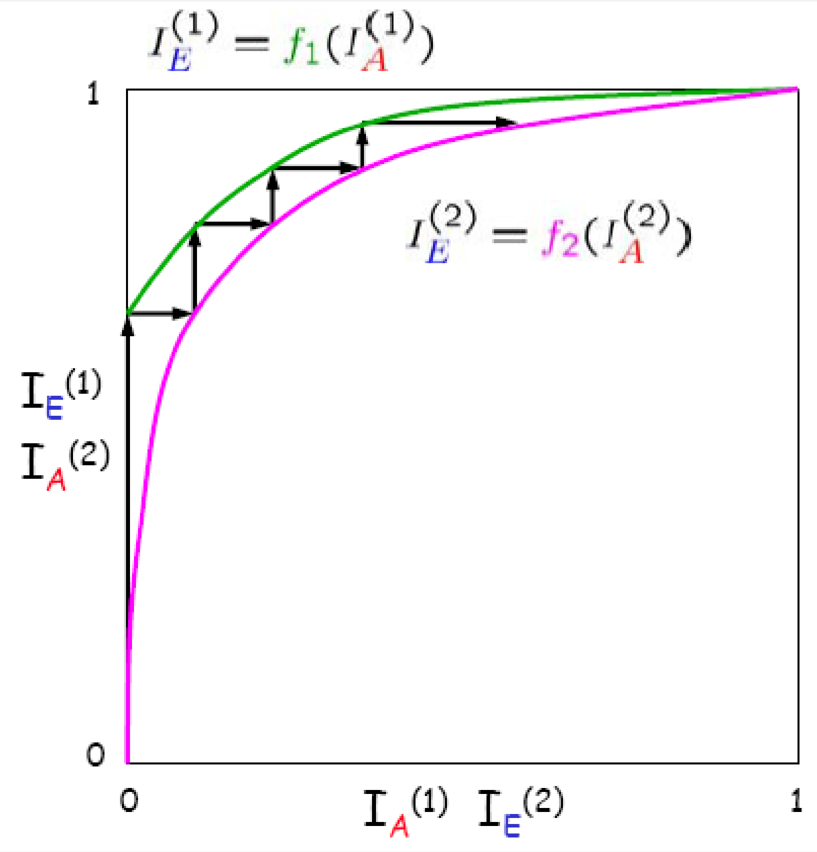


SNR=3dB, outer code rate 0.5, [13,15],

SNR=3dB, inner code rate 0.5, [7,5],

Combination of two transfer characteristics engenders EXIT Charts

Trajectory of iterative decoding



$$I_E^{(1)} = f_1(I_A^{(1)})$$

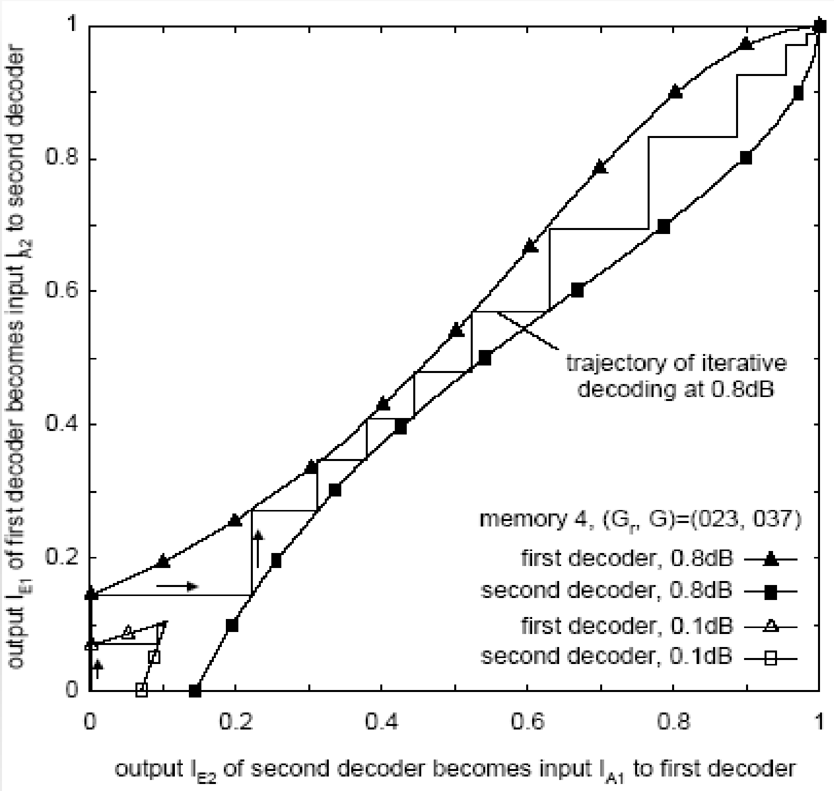
$$I_E^{(2)} \rightarrow I_A^{(1)}$$

$$I_E^{(2)} = f_2(I_A^{(2)})$$

$$I_E^{(1)} \rightarrow I_A^{(2)}$$

(J.Sayir, G.Lechner, "Theory and Design of Turbo and Related Codes", ftw.)

Exit Charts Features

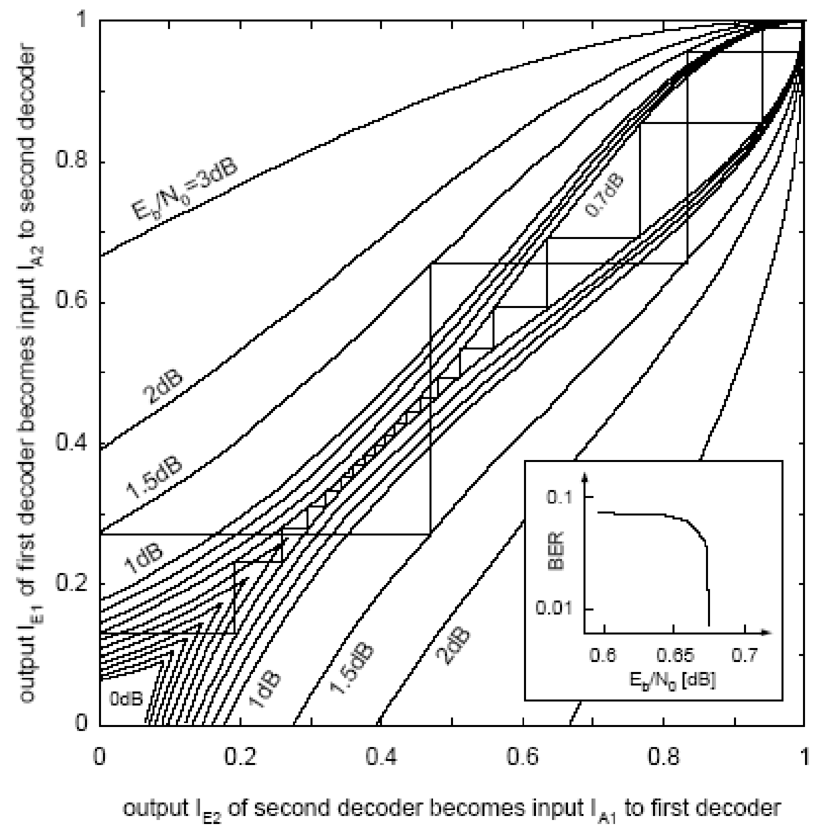


For $E_b/N_0 = 0.1dB$
Decoding trajectory got stuck early

For $E_b/N_0 = 0.8dB$
Decoding trajectory just "sneak through the bottleneck"

(Stephen ten Brink, "Convergence Behavior of Iteratively Decoded Parallel Concatenated Codes", IEEE Trans. Comm.Oct.2001)

Compare EXIT Charts with BER Charts



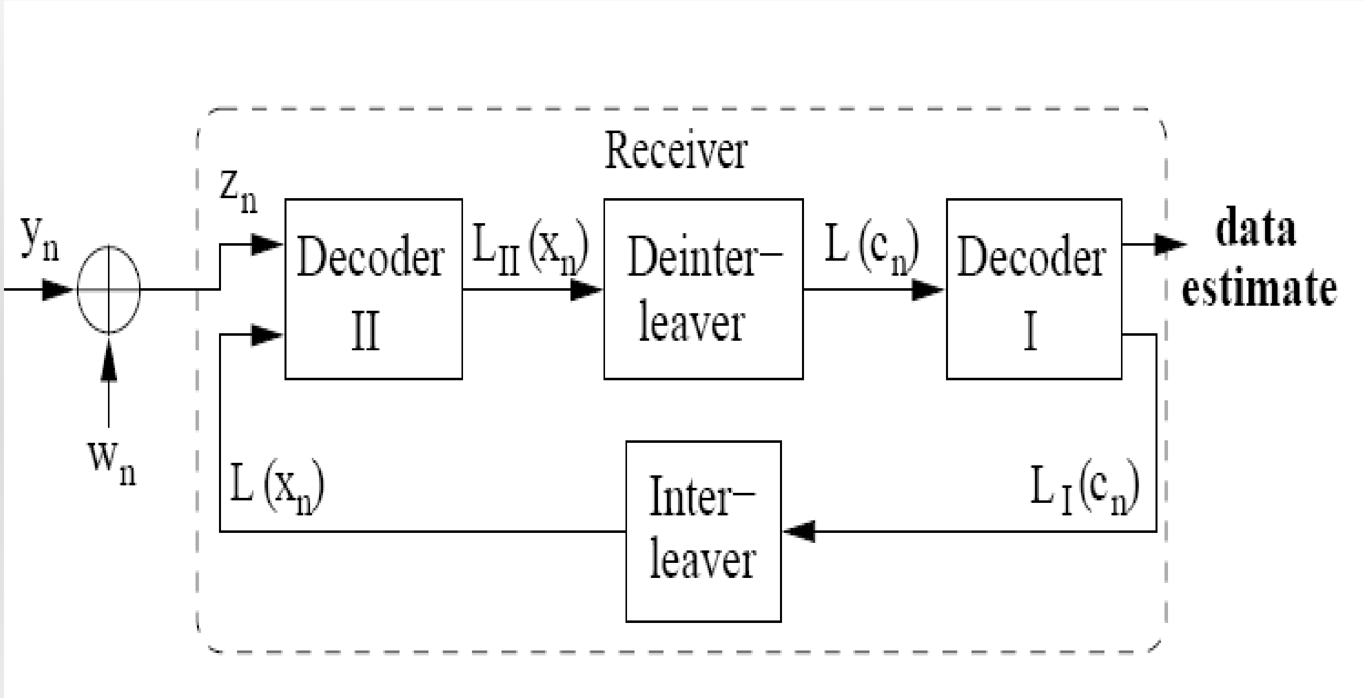
Pinch-off region, trajectory got stuck at low mutual information

Turbo cliff region is bottleneck region with slow convergence

BER floor region is the wide-open region with fast convergence

(Stephen ten Brink, “Convergence Behavior of Iteratively Decoded Parallel Concatenated Codes”, IEEE Trans. Comm.Oct.2001)

Iterative decoder used in serial Concatenated Codes



(J.Hagenauer, “The EXIT Chart- Introduction to Extrinsic Information Transfer in Iterative Processing”, EUSIPCO 2004)

Simplified Calculation of Mutual Information in serial concatenated structure

- **Two conditions to simplify:**

1. Probability density function is Symmetric

$$p(-y|x=+1) = p(y|x=-1)$$

2. Consistency condition

$$p(-y|x) = e^{-L_c xy} p(y|x)$$

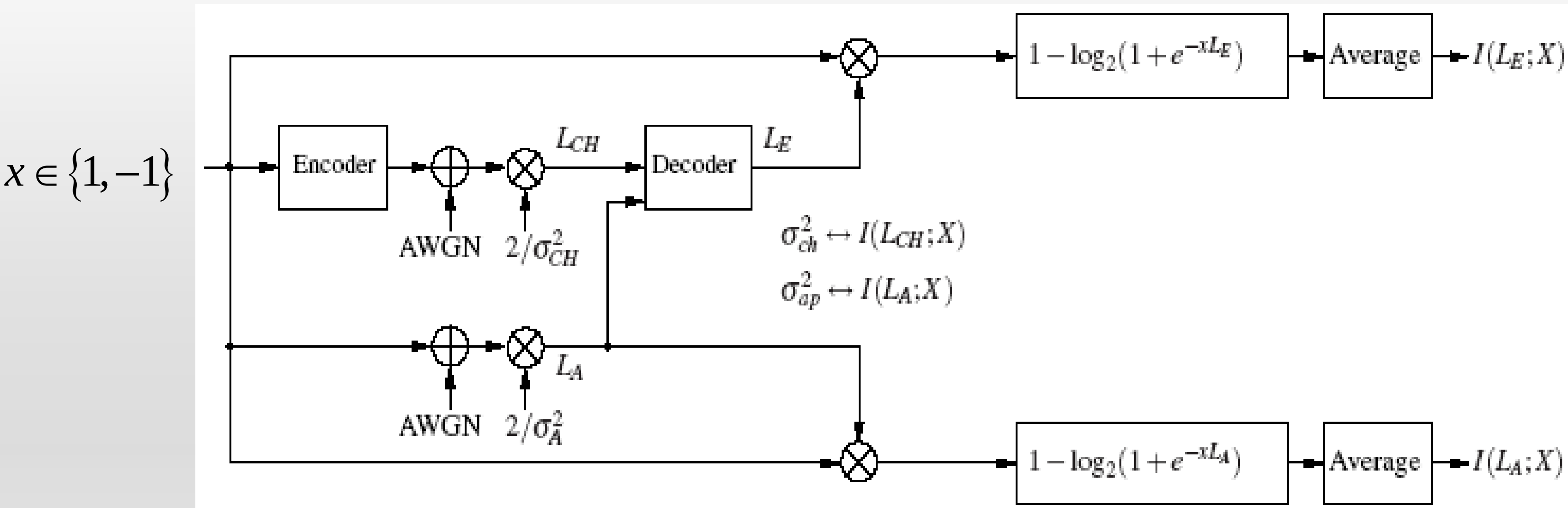
- **Simplify the calculation of mutual information**

$$I(L;X) = 1 - \int_{-\infty}^{+\infty} p\left(L|x=+1\right)\log_2\left(1+e^{-L}\right)dL$$

$$= 1 - E\left\{\log_2\left(1+e^{-L}\right)dL\right\}$$

$$p\left(L|x=+1\right) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\left(L-x\sigma^2/2\right)^2/2\sigma^2}$$

Measurement of EXIT Charts

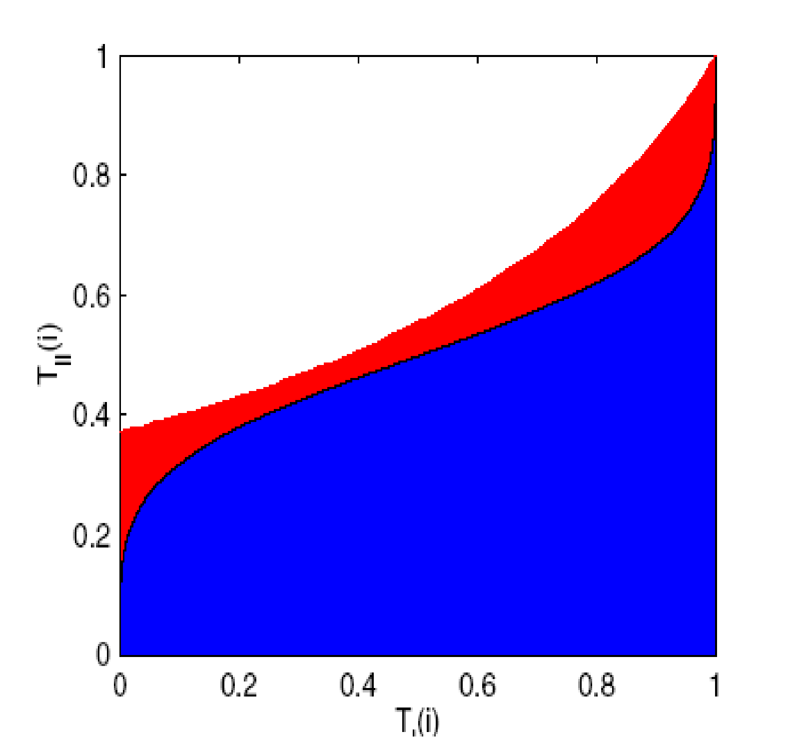


Output of lower branch $\longrightarrow$ horizontal axis

Output of upper branch $\longrightarrow$ vertical axis

(J.Hagenauer, "The EXIT Chart- Introduction to Extrinsic Information Transfer in Iterative Processing", EUSIPCO 2004)

Area Property of EXIT Charts



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Rate- Capacity properties for serial concatenated schemes

$$\int_0^1 T_{II}(i)\,di \approx C_{ui}$$

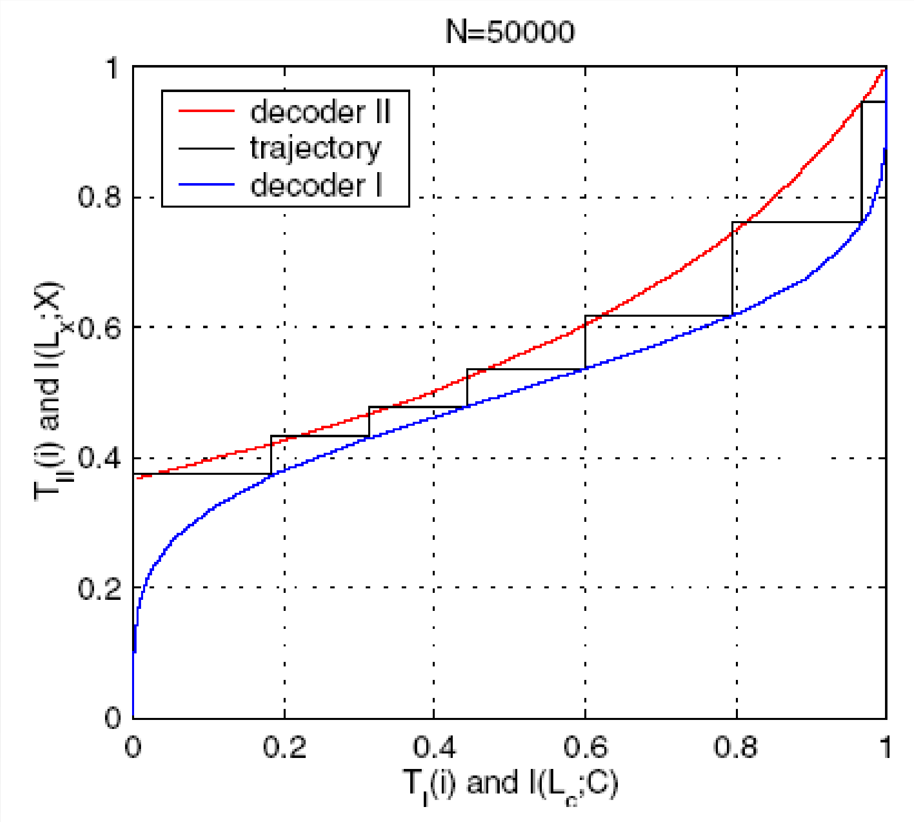
$$\int_0^1 T_I^{-1}(i)\,di \approx R_I$$

$$\int_0^1 \left(T_{II}(i) - T_I^{-1}(i)\right)di > 0 \Rightarrow R_I < C_{ui}$$

Code rate of outer code should be smaller than the capacity of inner channel

Code design is reduced to curve fitting

Simple example of EXIT Charts



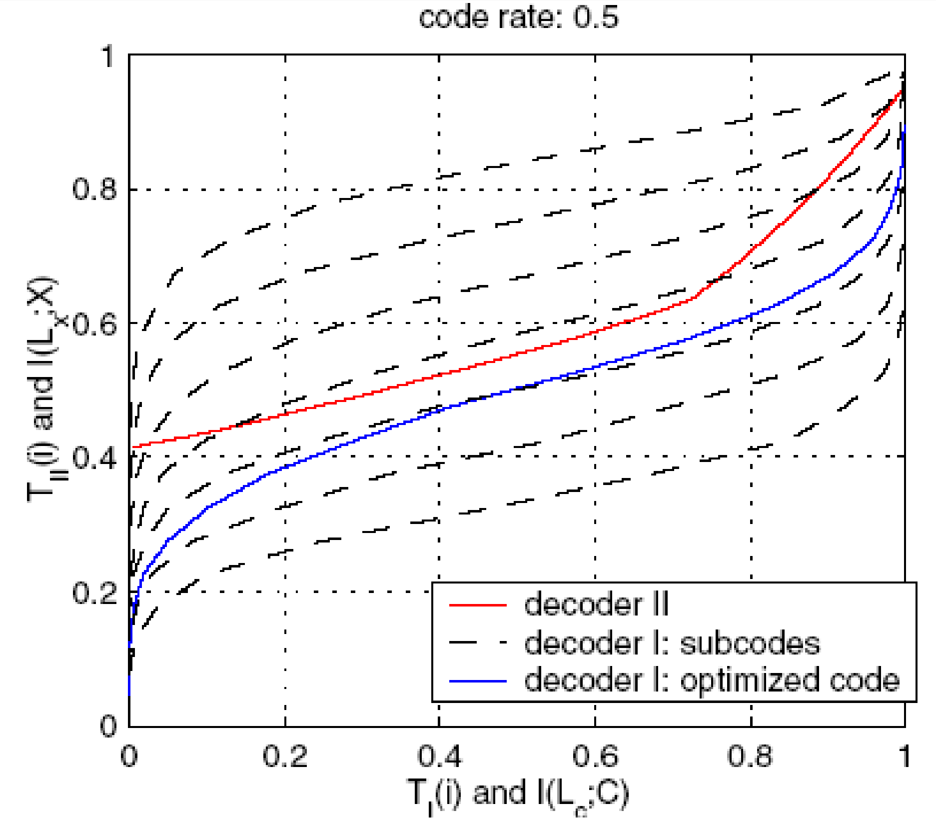
(J.Hagenauer, "The EXIT Chart- Introduction to Extrinsic Information Transfer in Iterative Processing", EUSIPCO 2004)

Simple Single Parity Check (SSPC) Codes

Serial concatenation with a DPSK modulator as inner code and an outer code M=2 convolutional code, R=1/2 at $E_b/N_0 = 1.5dB$

Trajectory slightly differs from Two EXIT curves due to limited depth of interleaver

Application of EXIT Charts



Multipath transmission as inner code

7 convolutional code as outer codes

Irregular codes perform good

(J.Hagenauer, "The EXIT Chart- Introduction to Extrinsic Information Transfer in Iterative Processing", EUSIPCO 2004)

Conclusion and Outlook

- **Based on mutual information to visualize the decoding trajectory of iterative decoding**
- **Came out of parallel concatenated structure and can be applied to serial concatenated structure**
- **It gives us some hints on designing good codes**
- **It can be utilized in a variety of fields (Irregular codes, LDPC, etc.)**

Thank you for your attention!

Questions & Discussions