Proseminar Randomisierte Methoden

A Simple Proof for the k–core Threshold

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February 21, 2006

Abstract

In dieser Arbeit wird XYZ vorgestellt. Dies hier ist eine kurze Zusammenfassung des Inhalts.

Contents

1 Introduction

Pittel, Spencer and Wormald have shown in [PSW96] that the probability for a random ...

... terminology and give a short exposition of the subject of phase transitions for random graphs in Section 2. Section 3 deals with the poles, note the discussion of the south pole in Subsection 3.2. In Section 4 we explain stuff concerning pictures in the text. Section 5 demonstrates the proof environments.

2 Phase Transitions and Random graphs

The Gibbs measure is the (appropriately defined) limit of the Boltzmann distribution for a given finite size Hamiltonian, when the volume goes to infinity. The system under consideration and thus the Hamiltonian is assumed to be homogenous, i.e. to 'look the same' from every point in space, which is usually achieved by assuming $\mathbb{Z}^d$ symmetry. Just think about ...

3 The Poles

3.1 The North Pole

Eine numerierte[1] Gleichung sieht folgendermassen aus:

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 7 & 5 & 3 \\ 1 & 2 & 8 \end{pmatrix} \tag{1}$$

Aus der Gleichung (1) sieht man, daß A eine Matrix ist.

Weiter könnten wir uns überlegen, Werte $a_1, a_2, \ldots, a_n$ miteinander zu multiplizieren und durch $\sum_{i=1}^{m} b_i$ zu teilen. Das sähe dann wohl so aus (nicht numeriert):

$$\frac{a_1 \cdot a_2 \cdots a_n}{b_1 + b_2 + \cdots + b_m} = 42$$

Auch mehrere Gleichungen untereinander sind manchmal nützlich:

$$u = 24 + 37 \tag{2}$$

$$u + v = 417 - u \tag{3}$$

Was folgt denn jetzt aus (1), (2) und (3)?

3.2 The South Pole

4 Figures in the Text

Look at Figure 1! Sometimes we want an empty figure, just to draw our own picture by hand (cf. Figure 2). However, note that there are tools for producing commutative diagrams in the AMS Latex package.

[1] Nach der Rechtschreibreform heißt es übrigens jetzt "'nummerierte"' statt "'numerierte"'.

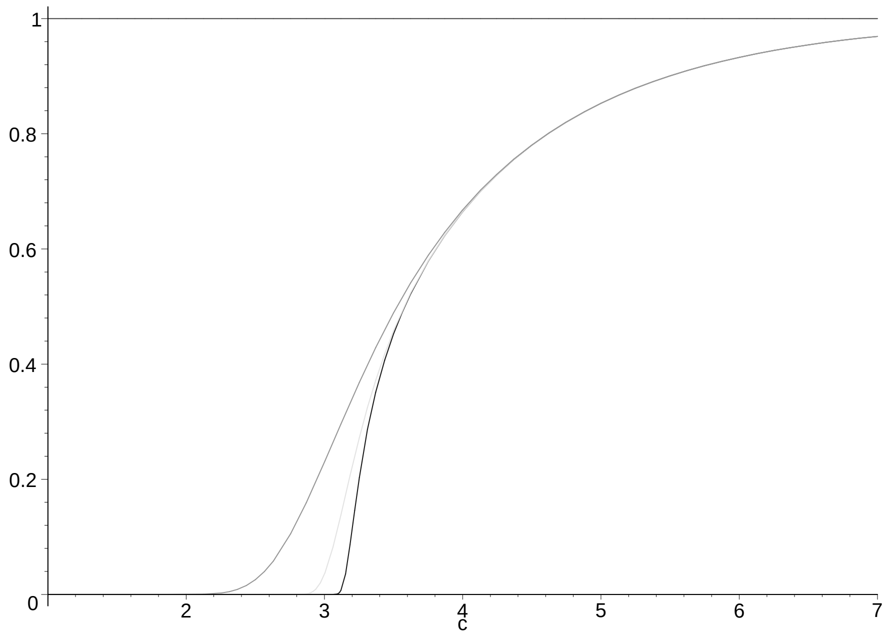

Figure 1: The curves $c \mapsto q_r^{\text{Caley}}(c)$ for $r = 5, 10, 15$.

Figure 2: An empty figure for your own drawings.

5 Some Definitions, Lemmas and Proofs

The branching–tree analogy as explained in [PSW96] inspired us to studying the random variable Z that counts the appearance of *Caley–trees* in a branching–tree. All balls and trees will be assumed to have a fixed radius r and all objects including the underlying probability spaces may (and will) depend on n, r and c, even if not explicitly stated.

Definition 5.1 *A Caley tree is a balanced tree such that all non–leaves have degree* 3*. It has* $3 \cdot 2^{r-1} - 2$ *inner nodes including the root, a total of* $3 \cdot 2^r - 2$ *nodes and* $3 \cdot 2^r - 3$ *edges.*

Note the following constructive asymptotic characterisation of Poisson–c–branching–trees:

- Start with the complete n–ary tree of radius r.

- Perform independent edge percolation with probability c/n.
- Delete all nodes that are not connected to the root.

From this $\mathbb{E}[Z]$ can be calculated by counting trees.

Lemma 5.2 *Given $c > 1$ for any $0 < \varepsilon < 0.029 < e^{-2/3}\frac{e^{5/3}}{c}$ in a GW branching tree with $Z_0 \equiv 1$ and $Z_1 \sim \mathrm{Po}_c$*

$$\mathbb{P}[Z_r \geq 1/2 \log(1/\varepsilon)(3c)^r] \leq r\varepsilon,$$

for all $r \in \mathbb{N}$.

Proof of Lemma 5.2. Proofs are not easy. □

Theorem 1 *Um den Seminarschein zu bekommen, muß man einen Vortrag halten und eine Ausarbeitung schreiben. Die Ausarbeitung muß rechtzeitig abgegeben werden.*

References

[PSW96] B. Pittel, J. Spencer, and N. Wormald. Sudden emergence of a giant k–core in a random graph. *Journal of Combinatorial Theory Series B*, 67:111–151, 1996.