Modelling and Computation of Dynamic Phase Transition of Liquids
- Compressible Flows with Cavitation -
Outline

Introduction

Important numbers

Physical effects

Modeling

CATUM

Cavitation Technische Universität München

Numerical results and validation

- Spherical body:

- Hydrofoil

- Prismatic body – cavitation erosion
Physics of cavitating flows

- Initially single-phase liquid fluid: $p_l \approx O(1-10^3 \text{ bar})$, $T \approx 300-400 \text{ K}$
  
- $p_v \approx O(10^{-2}-1 \text{ bar})$
  
- $\rho_v \approx O(10^{-2}-1 \text{ kg/m}^3)$
  
- $\rho_l \approx O(10^3 \text{ kg/m}^3)$

- Void fraction: $0 \leq \alpha \leq 1$

- Speed of sound: $c \approx O(1-10^3 \text{ m/s})$

- Strong variation of the Mach number: $M \approx O(0-10^1)$

- Dominating:
  - Strong density variation: $\rho_l/\rho_v \approx 10^4$
  - Strong variation of speed of sound: $c_l/c_{\text{min}} \approx 10^3$
  - Coexistence of compressible and weak compressible flow regimes
  - Formation of violent shocks in collapse region
  - Intense noise, vibration and erosion
Cavitation dynamics

1. Process
Depressurization - evaporation
increase of volume - 1 : 50000
displacement of liquid fluid
instability

2. Process
collapse
implosion of bubbles and cavitation patterns
violent shocks
erosion

Compressibility
local very low wave speed $c \leq 10 \text{ m/s}$
stiffness because of coupling
with regimes of $c \approx 1500 \text{ m/s}$
## Nucleation

<table>
<thead>
<tr>
<th>Homogeneous</th>
<th>Heterogeneous</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nucleus consists of molecules</td>
<td>Impurities like dissolved gas or crevices at walls or particles act as nucleus</td>
</tr>
<tr>
<td>Exclusive hom. nucleation allows high surface tension and highly metastable states</td>
<td></td>
</tr>
<tr>
<td>Only important with pure water</td>
<td>Dominant in most technical applications</td>
</tr>
</tbody>
</table>
Cavitation phenomena

Bubble and cloud cavitation

Sheet and cloud cavitation

Supercavitation

Vortex cavitation
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- Single bubble collapse
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Size of cavitation structures

Sphere diameter
$d_{\text{sphere}} = 1.5 \times 10^{-1} \text{ m}$

Size of a fluid element
$d_{\text{element}} = 5.2 \times 10^{-3} \text{ m}$
Two-phase flow properties via integral averages per cell

Vapor volume fraction per cell

\[ \alpha_{cell} = \frac{V_{vapor, cell}}{V_{cell}} \]

\[ \alpha_{cell1} = \alpha_{cell2} = \alpha_{cell3} \]

subgrid scale structures → integral average properties (FVM)

\[ \bar{\rho} = \frac{1}{V_{cell}} \int \rho \cdot dV \]

\[ \bar{\rho}u = \frac{1}{V_{cell}} \int \rho u \cdot dV \]

\[ \bar{\rho}E = \frac{1}{V_{cell}} \int \rho E \cdot dV \]

Stable thermodynamic conditions → constitutive relations (EOS) determine cell variables \( \bar{\rho}, \bar{T} \)

\[ \bar{\rho} = \bar{\rho}(\bar{p}, \bar{T}) \]

\[ \bar{e} = \bar{e}(\bar{p}, \bar{T}) \]
Thermodynamic Equilibrium Conditions - Substitute EOS

- “Equation of state” for liquid water: modified Tait “EOS” (thermal and caloric EOS for pure liquids)

\[
\bar{p}(\bar{\rho}, \bar{T}) = (B + p_{\text{sat}}(\bar{T})) \left( \frac{\bar{\rho}}{\rho_{l,\text{sat}}(\bar{T})} \right)^n - B
\]

\[
e_l(\bar{T}) = c_{vl} \cdot (\bar{T} - T_{\text{ref}}) + e_{l,\text{ref}}
\]

- EOS of pure water vapour: perfect gas law (thermal and caloric description of pure vapour)

\[
\bar{p}(\bar{\rho}, \bar{T}) = \bar{\rho} \cdot R_v \cdot \bar{T}
\]

\[
e_v(\bar{T}) = c_{vv} \cdot (\bar{T} - T_{\text{ref}}) + e_{v,\text{ref}} + l_v
\]

For water: \(B \approx 3.3 \cdot 10^8 \text{ Pa}, n \approx 7.15\), reference state \(T_{\text{ref}}\): expected mean temperature (293.15 K).

- EOS for saturated water/vapour: saturation conditions – Oldenbourg polynomials
(conditions for saturated mixture of water and water vapour for a void fraction \(\alpha\))

\[
\bar{p} = p_{\text{sat}}(\bar{T})
\]

\[
\bar{\rho} = \alpha \cdot \rho_{v,\text{sat}}(\bar{T}) + (1 - \alpha) \cdot \rho_{l,\text{sat}}(\bar{T})
\]

\[
\bar{\rho} e = \alpha \cdot \rho_{v,\text{sat}}(\bar{T}) \cdot e_v(\bar{T}) + (1 - \alpha) \cdot \rho_{l,\text{sat}}(\bar{T}) \cdot e_l(\bar{T}).
\]
Combined EOS contains relations for:
- pure water → modified Tait equation
- vapour phase → ideal gas law
- two-phase region → saturation conditions
Finite volume Method

Compressible, frictionless, unsteady flows -> Euler equations

Grid: structured hexagonal cells

Flux function: density based

Solver: mod. Riemann solver

2nd order accurate

explicit
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  - Single bubble collaps
  - Prismatic body – cavitation erosion
Numerical results – application: two-phase flow

3-D simulation of Branders experiment of cavitation around a sphere

\[ u_{in} = 10 \text{ m/s} \]
\[ T_{in} = 293 \text{ K} \]
\[ \sigma_{ref} = 0.8 \]

\[ d = 0.15 \text{ m} \]

\[ p_{out, \text{mix}} = 0.42 \text{ bar} \]

1.3 \cdot 10^6 \text{ cells}

1.2 \cdot 10^6 \text{ time steps}, \Delta t \approx 3 \cdot 10^{-7} \text{ s}

84 CPU (Operon) \rightarrow 80 \text{ h.}
Numerical results – application: two-phase flow

Comparison of two-phase structures experiment/simulation


Simulation CATUM: Isosurfaces $\alpha=0.05$, one instant in time.
Numerical results – 2-D hydrofoil

3-D simulation – span/channel width 0.3 m

\[ u_{in} = 30 \text{ m/s} \]
\[ T_{in} = 293.15 \text{ K} \]
\[ \sigma_{ref} = 1.0 \]

\[ p_{out, \text{mix}} = 4.5 \text{ bar} \]

2.4 \times 10^7 \text{ cells}
6 \times 10^5 \text{ time steps, } \Delta t \approx 5 \times 10^{-8} \text{ s,}
Numerical results – 2-D hydrofoil

2.4·10^7 cells,
6·10^5 time steps, Δt \approx 5·10^{-8} \text{ s},
96/192 CPU (lx64a Opteron) \rightarrow 500 \text{ h}

f_zyklus = 100 \text{ Hz},
Δt_{movie} = 3·10^{-2} \text{ s}
Numerical results – fragmentation of two-phase flow

2-D cavitation on 2-D hydrofoil

Effect of the spatial resolution on the structures of the vapor volume fraction $\alpha$. 
Numerical results – fragmentation of two-phase flow

2-D cavitation on 2-D hydrofoil

Effect of the spatial resolution on the instantaneous maximum loads – pressure footprint over one cycle
Numerical results – fragmentation of two-phase flow

2-D cavitation on 2-D hydrofoil

Effect of the spatial resolution on the instantaneous maximum loads – pressure footprint over one cycle, **zooms of previous pictures**
2-D hydrofoil – maximum pressure

Collaps induced maximum pressure on the suction side - \( p_{\text{max}} \approx 2000 \) bar

Analysed time: one period with \( \Delta t_{\text{zyklus}} = 10^{-2} \) s
Cavitation erosion

Kuiper, G. - MARIN Maritime Research Institute - The Netherlands
Driving mechanisms of cavitation erosion
Single bubble collapse with wall interaction

Initial radius $R_0=0.5$ mm, time step $\Delta t_{\text{CFD}}=6.0 \cdot 10^{-9}$ s, collapse time $1.7 \cdot 10^{-5}$ s,
Initial pressures $p_{\text{liquid}}=10.0$ bar, $p_{\text{bubble}}=0.023$ bar, $T=293$ K, water/vapor

Simulation CATUM
Numerical results – Erosive two-phase flow

3-D simulation of the “Obernach-experiment” on cavitation erosion

$u_{in}=11 \, \text{m/s}$
$T_{in}=300 \, \text{K}$
$\sigma_{ref}=1.8$

$P_{out,\text{mix}}=1.12 \, \text{bar}$

3.1$\times10^6 \, \text{cells}$
$10^6 \, \text{time steps}, \Delta t \approx 3\times10^{-7} \, \text{s}$

64 CPU (SGI AltixBx2) $\rightarrow$ 240 h.
Numerical results – Erosive two-phase flow

Dynamic phase-transition and related pressure field

Top view: Two-phase regions, $\Delta t_{\text{Movie}}=0.17$ s.

Perspective view: Two-phase regions and static pressure at the walls, $\Delta t_{\text{Movie}}=0.17$ s.
Numerical results – Erosive two-phase flow

Comparison of two-phase structures experiment/simulation


Simulation CATUM: Isosurfaces $\alpha=0.01$, one instant in time.
Numerical results – Erosive two-phase flow

Fragmentation of two-phase structure, collapse, shock formation

$\Delta t_{1 \rightarrow 2} = 1.17 \cdot 10^{-4}$ s

$\Delta t_{2 \rightarrow 3} = 0.58 \cdot 10^{-4}$ s

$p_{\text{max}} = 65$ bar
Numerical results – Erosive two-phase flow

Areas of intense erosion (experiment) - maximum pressures (simulation)


Simulation CATUM: Collapse induced maximum pressure at the bottom wall of the numerical test-section, analysis interval 0.058 seconds. Stars indicate the barycenters (experimental) of the erosion ares.
Discussion