Suppose you want to develop a data structure with:

- **Insert**$(x)$: insert element $x$.
- **Search**$(k)$: search for element with key $k$.
- **Delete**$(x)$: delete element referenced by pointer $x$.
- **find-by-rank**$(\ell)$: return the $k$-th element; return “error” if the data-structure contains less than $k$ elements.

Augment an existing data-structure instead of developing a new one.
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How to augment a data-structure

1. choose an underlying data-structure
2. determine additional information to be stored in the underlying structure
3. verify/show how the additional information can be maintained for the basic modifying operations on the underlying structure.
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Goal: Design a data-structure that supports insert, delete, search, and find-by-rank in time $O(\log n)$.

1. We choose a red-black tree as the underlying data-structure.
2. We store in each node $v$ the size of the sub-tree rooted at $v$.
3. We need to be able to update the size-field in each node without asymptotically affecting the running time of insert, delete, and search. We come back to this step later...
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4. How does find-by-rank work?
   Find-by-rank($k$) := Select(root, $k$) with

   
   **Algorithm 15 Select($x$, $i$)**
   
   1: **if** $x = \text{null}$ **then** return error
   2: **if** left[$x$] $\neq \text{null}$ **then** $r \leftarrow \text{left}[x].\text{size} + 1$ **else** $r \leftarrow 1$
   3: **if** $i = r$ **then** return $x$
   4: **if** $i < r$ **then**
   5: \quad return Select(left[$x$], $i$)
   6: **else**
   7: \quad return Select(right[$x$], $i - r$)
Select \((x, i)\)

Find-by-rank:

- decide whether you have to proceed into the left or right sub-tree
- adjust the rank that you are searching for if you go right
Select($x, i$)

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Select\((x, i)\)

Select\((13, 14)\)

Find-by-rank:
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Select\((x, i)\)

Select\((21, 5)\)

Find-by-rank:

- decide whether you have to proceed into the left or right sub-tree
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Select$(x, i)$

**Select**$(16, 5)$

Find-by-rank:
- decide whether you have to proceed into the left or right sub-tree
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**Select**($x$, $i$)

**Select**($19$, 3)

Find-by-rank:
- decide whether you have to proceed into the left or right sub-tree
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Select \((x, i)\)

Select \((20, 1)\)

Find-by-rank:

- decide whether you have to proceed into the left or right sub-tree
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Goal: Design a data-structure that supports insert, delete, search, and find-by-rank in time $\mathcal{O}(\log n)$.

3. How do we maintain information?

Search($k$): Nothing to do.

Insert($x$): When going down the search path increase the size field for each visited node. Maintain the size field during rotations.

Delete($x$): Directly after splicing out a node traverse the path from the spliced out node upwards, and decrease the size counter on every node on this path. Maintain the size field during rotations.
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Rotations

The only operation during the fix-up procedure that alters the tree and requires an update of the size-field:

\[ \text{LeftRotate}(x) \]
\[ \text{RightRotate}(z) \]

The nodes \( x \) and \( z \) are the only nodes changing their size-fields. The new size-fields can be computed \textit{locally} from the size-fields of the children.