Technique 3: The Primal Dual Method

The previous two rounding algorithms have the disadvantage that it is necessary to solve the LP. The following method also gives an f-approximation without solving the LP.

For estimating the cost of the solution we only required two properties.

1. The solution is dual feasible and, hence,

$$\sum_{u} y_{u} \le \operatorname{cost}(x^{*}) \le \operatorname{OPT}$$

where x^* is an optimum solution to the primal LP.

2. The set *I* contains only sets for which the dual inequality is tight.

Of course, we also need that *I* is a cover.

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Algorithm 1 PrimalDual
1: $y \leftarrow 0$
2: $I \leftarrow \emptyset$
3: while exists $u \notin \bigcup_{i \in I} S_i$ do
4: increase dual variable y_i until constraint for some
new set S_ℓ becomes tight
5: $I \leftarrow I \cup \{\ell\}$

