# Praktikum Algorithmenentwurf 

Due date: Monday, 18th November 2013, 14:00

## Aufgabe 1 (Weighted Matchings weightedmatching)

Let $G=(V, E)$ be an undirected bipartite graph such that $V=V_{1} \cup V_{2}, V_{1} \cap V_{2}=\emptyset$ and every edge is adjacent to a node of $V_{1}$ and a node of $V_{2}$. Let $w(e) \in \mathbb{Z}$ be the edge weight of edge $e \in E$. Notice that negative edge weights are allowed! Implement an efficient algorithm that computes in time $O\left(|V|^{2} \log |V|+|V| \cdot|E|\right)$ a maximum matching (i.e. a matching having maximum cardinality) of $G$ which has maximum weight over all maximum matchings of $G$. (The weight of a matching equals the sum of the weights of its edges.) Extend the algorithm in such a way that the user can choose if a maximum matching with minimum weight or a maximum matching with maximum weight is computed. Your program should output the weight of the computed matching at the end.

Use your implementation of Dijkstra of exercise sheet 3 as a subroutine if possible.
Utilize the visualization capabilities of GraphWin to vividly visualize how the algorithm works. One possibility is to open a second GraphWin window (beside the one showing the input graph) displaying the modified graph $G^{\prime}$ with directed edges and the additional vertices $s$ and $t$.

## Hints

You can use the graphs wbipartite1.gw to wbipartite4.gw as inputs for your algorithm. In this graphs all nodes of $V_{1}$ contain the user label " 1 " while the nodes of $V_{2}$ contain the user label " 2 ". The weights of the edges are stored in the user-labels of the edges. All edge weights are numbers between -2 and 17 , and your program can visualize these weights well by setting the edge width to the weight of the edge (for edges with negative weights dashed lines can be used.)
The following table shows the weight of a maximum matching with minimal and maximal weight, respectively.

|  | wbipartite1.gw | wbipartite2.gw | wbipartite3.gw | wbipartite4.gw |
| :--- | :---: | :---: | :---: | :---: |
| minimal | 29 | -2 | 10 | 46 |
| maximal | 33 | -1 | 22 | 81 |

