## Automata and Formal Languages

Due January 20, 2015 before class!

## Exercise 1 (Büchi-Automata - 10 points)

Given a Büchi automaton $A$ and finite words $u, v$, decide whether $A$ accepts the $\omega$-word $u v^{\omega}$

## Exercise 2 ( $\omega$-expressions I)

Let $\Sigma=\{a, b, c\}$. Give an $\omega$-expression for each of the following languages:
(a) $L_{1}=\{w \mid ' a b$ ' occurs only finitely often in $w\}$
(b) $L_{2}=\{w \mid$ every ' $a$ ' is immediately followed by a ' $b$ ' $\}$
(c) $L_{3}=\{w \mid$ every ' $b$ ' is preceded by an ' $a$ ' $\}$

## Exercise 3 ( $\omega$-expressions II - 10 points)

Give Büchi- and Muller-automata for the following languages:
(a) $r_{1}=\left(a^{*} b\right)^{\omega}$
(b) $r_{2}=\left(010^{*}\right)^{\omega}+1^{\omega}$
(c) $r_{3}=(a b+b c+a)^{\omega}$

## Exercise 4 (Ranking - 10 points)

Consider the following Büchi- automaton $B$ representing the $\omega$-words over $\Sigma=\{a, b\}$ having only finitely many as:

(a) Sketch $\operatorname{dag}\left(a b a b^{\omega}\right)$ and $\operatorname{dag}\left((a b)^{\omega}\right)$.
(b) Consider the ranking $r$ defined as $r\left(<q_{0}, i>\right)=1$ and $r\left(<q_{1}, i>\right)=0$ for all $i \in \mathbb{N}$. Is $r$ an odd ranking for the two dags from (a)?
(c) Show:

Ranking $r$ defined in (b) is odd $\Leftrightarrow w \notin \mathcal{L}(B)$

