Technische Universität München Fakultät für Informatik Lehrstuhl für Effiziente Algorithmen (LEA) Prof. Dr. Ernst W. Mayr Moritz Fuchs

# Automata and Formal Languages

Due January 20, 2015 before class!

### Exercise 1 (Büchi-Automata - 10 points)

Given a Büchi automaton A and finite words u,v, decide whether A accepts the  $\omega\text{-word}\;uv^\omega$ 

### Exercise 2 ( $\omega$ -expressions I)

Let  $\Sigma = \{a, b, c\}$ . Give an  $\omega$ -expression for each of the following languages:

(a)  $L_1 = \{w \mid ab \text{ occurs only finitely often in } w\}$ 

(b)  $L_2 = \{w \mid \text{every '}a' \text{ is immediately followed by a '}b'\}$ 

(c)  $L_3 = \{w \mid \text{every 'b' is preceded by an 'a'} \}$ 

## Exercise 3 ( $\omega$ -expressions II - 10 points)

Give Büchi- and Muller-automata for the following languages:

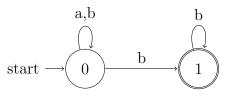
(a) 
$$r_1 = (a^*b)^{\omega}$$

(b) 
$$r_2 = (010^*)^\omega + 1^\omega$$

(c)  $r_3 = (ab + bc + a)^{\omega}$ 

### Exercise 4 (Ranking - 10 points)

Consider the following Büchi- automaton B representing the  $\omega$ -words over  $\Sigma = \{a, b\}$  having only finitely many as:



- (a) Sketch dag $(abab^{\omega})$  and dag $((ab)^{\omega})$ .
- (b) Consider the ranking r defined as  $r(\langle q_0, i \rangle) = 1$  and  $r(\langle q_1, i \rangle) = 0$  for all  $i \in \mathbb{N}$ . Is r an odd ranking for the two dags from (a)?
- (c) Show:

Ranking r defined in (b) is odd 
$$\Leftrightarrow w \notin \mathcal{L}(B)$$