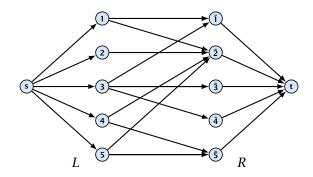
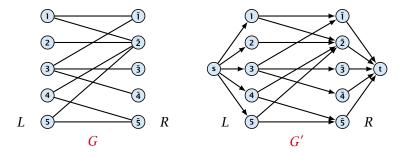
17 Bipartite Matching via Flows

- ▶ Input: undirected, bipartite graph $G = (L \uplus R \uplus \{s, t\}, E')$.
- Direct all edges from L to R.
- Add source *s* and connect it to all nodes on the left.
- Add *t* and connect all nodes on the right to *t*.
- All edges have unit capacity.



Max cardinality matching in $G \leq$ value of maxflow in G'

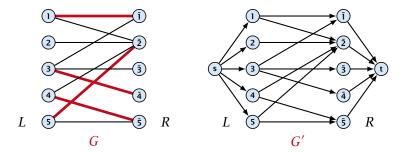
- Given a maximum matching *M* of cardinality *k*.
- Consider flow *f* that sends one unit along each of *k* paths.
- f is a flow and has cardinality k.





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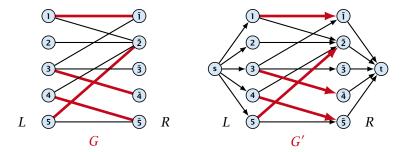


17 Bipartite Matching via Flows

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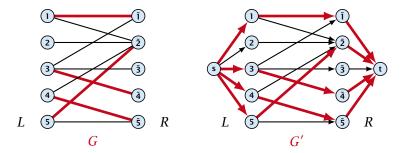


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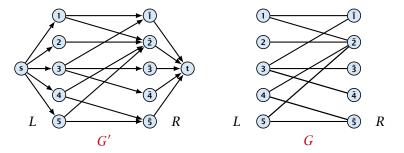


17 Bipartite Matching via Flows

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Max cardinality matching in $G \ge$ value of maxflow in G'

- Let f be a maxflow in G' of value k
- Integrality theorem $\Rightarrow k$ integral; we can assume f is 0/1.
- Consider M= set of edges from L to R with f(e) = 1.
- Each node in *L* and *R* participates in at most one edge in *M*.
- |M| = k, as the flow must use at least k middle edges.

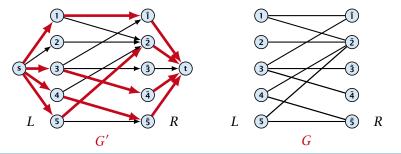




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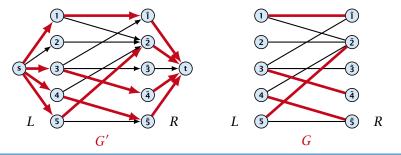


17 Bipartite Matching via Flows

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17 Bipartite Matching via Flows

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17 Bipartite Matching via Flows

Which flow algorithm to use?

- Generic augmenting path: $\mathcal{O}(m \operatorname{val}(f^*)) = \mathcal{O}(mn)$.
- Capacity scaling: $\mathcal{O}(m^2 \log C) = \mathcal{O}(m^2)$.

