Baseball Elimination

team	wins	losses	remaining games			
i	w_i	ℓ_i	Atl	Phi	NY	Mon
Atlanta	83	71	−	1	6	1
Philadelphia	80	79	1	−	0	2
New York	78	78	6	0	−	0
Montreal	77	82	1	2	0	−

Which team can end the season with most wins?

▶ Montreal is eliminated, since even after winning all remaining games there are only 80 wins.
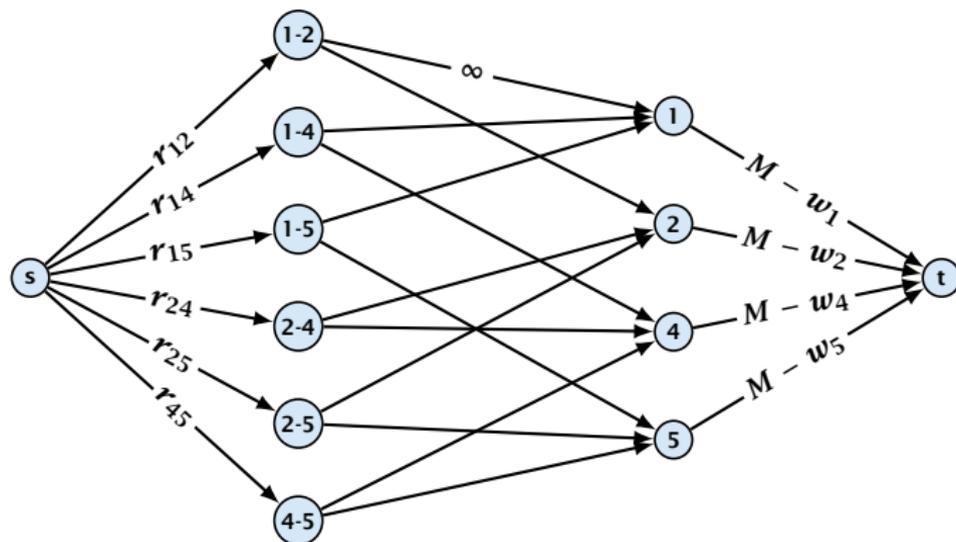
▶ But also Philadelphia is eliminated. Why?

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Formal definition of the problem:

- Given a set S of teams, and one specific team $z \in S$.
- Team x has already won w_x games.
- Team x still has to play team y, r_{xy} times.
- Does team z still have a chance to finish with the most number of wins.

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Flow network for $z = 3$. M is number of wins Team 3 can still obtain.

Idea. Distribute the results of remaining games in such a way that no team gets too many wins.

Certificate of Elimination

Let $T \subseteq S$ be a subset of teams. Define

$$w(T) := \sum_{i \in T} w_i, \qquad r(T) := \sum_{i,j \in T, i<j} r_{ij}$$

wins of teams in T

remaining games among teams in T

If $\frac{w(T)+r(T)}{|T|} > M$ then one of the teams in T will have more than M wins in the end. A team that can win at most M games is therefore eliminated.

Theorem 1

A team z is eliminated if and only if the flow network for z does not allow a flow of value $\sum_{ij \in S \setminus \{z\}, i<j} r_{ij}$.

Proof ($\Leftarrow$)

- ▸ Consider the mincut A in the flow network. Let T be the set of team-nodes in A.

- ▸ If for node x-y not both team-nodes x and y are in T, then x-$y \notin A$ as otw. the cut would cut an infinite capacity edge.

- ▸ We don't find a flow that saturates all source edges:

$$r(S \setminus \{z\}) > \mathrm{cap}(A, V \setminus A)$$
$$\geq \sum_{i<j:\, i \notin T \vee j \notin T} r_{ij} + \sum_{i \in T} (M - w_i)$$
$$\geq r(S \setminus \{z\}) - r(T) + |T|M - w(T)$$

- ▸ This gives $M < (w(T) + r(T))/|T|$, i.e., z is eliminated.

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Proof (⇒)

- ▶ Suppose we have a flow that saturates all source edges.
- ▶ We can assume that this flow is integral.
- ▶ For every pairing x-y it defines how many games team x and team y should win.
- ▶ The flow leaving the team-node x can be interpreted as the additional number of wins that team x will obtain.
- ▶ This is less than $M - w_x$ because of capacity constraints.
- ▶ Hence, we found a set of results for the remaining games, such that no team obtains more than M wins in total.
- ▶ Hence, team z is not eliminated.