

Project Selection

Project selection problem:

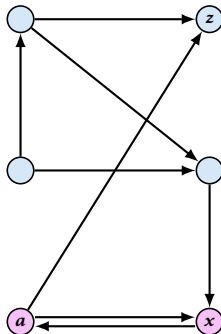
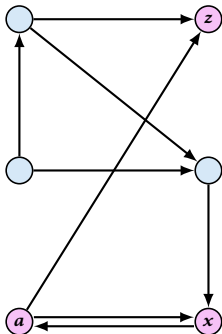
- ▶ Set P of possible projects. Project v has an associated profit p_v (can be positive or negative).
- ▶ Some projects have requirements (taking course EA2 requires course EA1).
- ▶ Dependencies are modelled in a graph. Edge (u, v) means “can’t do project u without also doing project v .”
- ▶ A subset A of projects is **feasible** if the prerequisites of every project in A also belong to A .

Goal: Find a feasible set of projects that maximizes the profit.

Project Selection

The prerequisite graph:

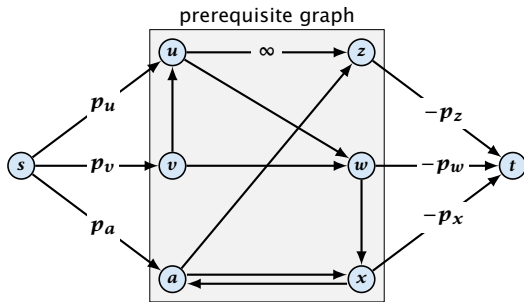
- ▶ $\{x, a, z\}$ is a feasible subset.
- ▶ $\{x, a\}$ is infeasible.



Project Selection

Mincut formulation:

- ▶ Edges in the prerequisite graph get infinite capacity.
- ▶ Add edge (s, v) with capacity p_v for nodes v with positive profit.
- ▶ Create edge (v, t) with capacity $-p_v$ for nodes v with negative profit.



Theorem 2

A is a mincut if $A \setminus \{s\}$ is the optimal set of projects.

Proof.

▶ A is feasible because of capacity infinity edges.

$$\text{cap}(A, V \setminus A) = \sum_{v \in \bar{A}: p_v > 0} p_v + \sum_{v \in A: p_v < 0} (-p_v)$$

$$= \sum_{v: p_v > 0} p_v - \sum_{v \in A} p_v$$

For the formula we define $p_s := 0$.

The step follows by adding $\sum_{v \in A: p_v > 0} p_v - \sum_{v \in A: p_v > 0} p_v = 0$.

Note that minimizing the capacity of the cut $(A, V \setminus A)$ corresponds to maximizing profits of projects in A .

