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Problem Set 08
December 1st, 2014

Parallel Algorithms

Due date: December 8th, 2014 **before** *class!*

Problem 1 (10 Points)

The *1-color minimization* problem is described as follows: Given a set of n processors P_i each having a color $x_i \in \{0,1\}$ in its local memory, each processor P_i is supposed to compute the value $a_i \in \{0,1\}$ such that $a_i = 1$ if and only if P_i is the lowest-indexed processor with $x_i = 1$. On problem set 1 we have seen how to solve this problem in $\mathcal{O}(1)$ time on the common CRCW when no restrictions are placed on the size of the shared memory. Show that the 1-color minimization problem can be solved in $\mathcal{O}\left(\frac{\log n}{\log(m+1)}\right)$ time on common(m).

Problem 2 (10 Points)

Show that common(m) can simulate one step of priority(m) in $\mathcal{O}(\log n)$ steps.

Problem 3 (10 Points)

The purpose of this exercise is to show that the Boolean OR function of n variables can be computed by an n-processor CREW PRAM in $\leq \log_{2.618} n + \mathcal{O}(1)$ steps, which is less than $\log_2 n$.

Let the input bits $x_1, \ldots, x_n$ be stored in $M(1), \ldots, M(n)$ of the global memory, and let $n = F_{2T+1}$, where F_i is the ith Fibonacci number with $F_0 = 0, F_1 = 1$, and $F_{m+2} = F_{m+1} + F_m$ for $m \geq 0$. Each processor P_i uses two variables y_i and t, initially set to 0. The algorithm executed by P_i is the following:

- **if** $i + F_{2t} \leq n$ **then** $y_i \leftarrow y_i \vee M(i + F_{2t})$
- **if** $(i > F_{2t+1}$ **and** $y_i = 1)$ **then** $M(i - F_{2t+1}) \leftarrow 1$

1. Show that, just before step t, we have $y_i = x_i \vee x_{i+1} \vee \cdots \vee x_{i+F_{2t}-1}$ and $M(i) = x_i \vee x_{i+1} \vee \cdots \vee x_{i+F_{2t+1}-1}$ for $1 \leq i \leq n$.
2. Deduce that the algorithm uses at most $\log_{2.618} n + \mathcal{O}(1)$ steps.