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Randomized Algorithms

Exercise Sheet 11

Due: January 19, 2015

Exercise 1: (10 points)
In the BIN PACKING problem, we are given a set of items $I = \{1, 2, \ldots, n\}$ which have to be to packed into bins. Each item $i \in I$ has a volume $0 \leq v_i \leq 1$ and it has to be assigned entirely to a single bin. The capacity of a bin is equal to 1. We would like to pack the items in a way that the total volume of the items stored in a bin does not exceed its capacity. Our objective is to minimize the number of used bins. Formulate the BIN PACKING problem as an integer linear program.

Exercise 2: (10 points)
Consider a graph $G = (V, E)$ with n vertices. A *vertex coloring* of G is an assignment of colors to the vertices in V so that any two adjacent vertices are not assigned the same color. In the VERTEX COLORING problem, the objective is to find a vertex coloring using a minimum number of different colors. This number is known as the chromatic number of G and it is usually denoted by $\chi(G)$. Formulate the problem of finding the chromatic number of a graph (and the corresponding vertex coloring) as an integer linear program.

Exercise 3: (10 points)
Let K_n be a complete graph with n vertices. A *Hamilton cycle* of a graph G is a cycle which visits every vertex of G exactly once. Moreover, for a given edge coloring of a graph G, we say that a path is *monochromatic* if all its edges have the same color. Show that, for any $n \geq 3$, in any edge coloring of K_n with 2 colors, there exists a Hamilton cycle which is the union of two monochromatic paths. Note that one of the two paths may contain no edges and, in this case, the whole Hamilton cycle is monochromatic. *Hint: You do not need a probabilistic argument.*

Exercise 4: (10 points)

- Prove that, for every integer n, there exists a coloring of the edges of the complete graph K_n with two colors so that the total number of monochromatic K_4 cliques is at most $\binom{n}{4}(\frac{1}{2})^5$.
  Hint: Use an expectation argument.

- Give a randomized algorithm for finding a coloring with at most $\binom{n}{4}(\frac{1}{2})^5$ monochromatic K_4 cliques with an expected running time polynomial in n.