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November 17, 2014

Randomized Algorithms

Exercise Sheet 6

Due: November 24, 2014

Exercise 1: (10 points)
We throw n balls into n bins independently and uniformly at random. Show that the expected number of bins with exactly one ball is upper bounded by $\simeq \frac{n}{e}$.

Hint: Use the approximation $(1 - \frac{1}{n})^n \simeq \frac{1}{e}$.

Exercise 2 (10 points)
Consider a parallel computer with n processors and n memory cells. In one computational step, each processor makes a request for one memory cell chosen uniformly at random. A memory cell can be requested by at most two processors in one step. If it is requested by more than two processors, then none of these requests is served. Show that the expected number of processors whose requests are served is $\simeq \frac{2n}{e}$, for large n.

Exercise 3 (10 points)
We throw m balls into n bins independently and uniformly at random. Use Markov's and Chebyshev's inequalities in order to compute upper bounds on the probability that a bin contains at least k balls. Compare these bounds when $m = n$.

Exercise 4 (10 points)
Consider an algorithm A for which all we know is that it takes as input a string of n bits and that its expected running time is $O(f(n))$ if the input bits are chosen independently and uniformly at random. By using Markov's inequality, what can we say about the worst-case running time of A for inputs of size n?