Let I denote the solution obtained by the first rounding algorithm and I' be the solution returned by the second algorithm. Then

$$I \subseteq I'$$
.

This means I' is never better than I.

- ▶ Suppose that we take  $S_i$  in the first algorithm. I.e.,  $i \in I$ .
- ▶ This means  $x_i \ge \frac{1}{f}$ .
- ▶ Because of Complementary Slackness Conditions the corresponding constraint in the dual must be tight.
- $\blacktriangleright$  Hence, the second algorithm will also choose  $S_i$ .

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# **Technique 3: The Primal Dual Method**

#### Algorithm 1 PrimalDual

- 1:  $v \leftarrow 0$
- 2: *I* ← ∅
- 3: **while** exists  $u \notin \bigcup_{i \in I} S_i$  **do**
- increase dual variable  $y_u$  until constraint for some new set  $S_{\ell}$  becomes tight
- $I \leftarrow I \cup \{\ell\}$

### **Technique 3: The Primal Dual Method**

The previous two rounding algorithms have the disadvantage that it is necessary to solve the LP. The following method also gives an f-approximation without solving the LP.

For estimating the cost of the solution we only required two properties.

1. The solution is dual feasible and, hence,

$$\sum_{u} y_{u} \le \operatorname{cost}(x^{*}) \le \operatorname{OPT}$$

where  $x^*$  is an optimum solution to the primal LP.

**2.** The set *I* contains only sets for which the dual inequality is tight.

Of course, we also need that *I* is a cover.

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13.3 Primal Dual Technique

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## **Technique 4: The Greedy Algorithm**

#### Algorithm 1 Greedy

1: *I* ← Ø

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- 2:  $\hat{S}_i \leftarrow S_i$  for all j
- 3: while I not a set cover do
- $\ell \leftarrow \arg\min_{j:\hat{S}_j \neq 0} \frac{w_j}{|\hat{S}_j|}$
- $I \leftarrow I \cup \{\ell\}$
- $\hat{S}_i \leftarrow \hat{S}_i S_\ell$  for all j

In every round the Greedy algorithm takes the set that covers remaining elements in the most cost-effective way.

We choose a set such that the ratio between cost and still uncovered elements in the set is minimized.