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## Complexity Theory

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*Due date: June 15, 2015 before class!*

### Problem 1 (10 Points)

Show that there is a language  $B \in \mathbf{EXP}$  such that  $\mathcal{NP}^B \neq \mathcal{P}^B$ .

### Problem 2 (10 Points)

Geography is a game where players take turns naming cities. Each city chosen must begin with the final letter of the previous chosen city. Repetition is not allowed. The game starts with an arbitrary starting city and a player loses if he or she is unable to continue. Define GENERALIZED GEOGRAPHY as a generic graph problem and show that it is **PSPACE**-complete.

### Problem 3 (10 Points)

Given a context-sensitive grammar  $G$  and a word  $x \in \Sigma^*$ , Context-Sensitive Recognition (CSR) is the problem of deciding membership of  $x$  in  $L(G)$ , the language that is generated by  $G$ . In fact, this problem can be solved in **NSPACE**( $n$ ) by guessing the right application of  $G$ .

Show that CSR is **PSPACE**-complete. (Note that you do not have to know more about context-sensitive grammars.)

*Hint:* You may use the fact that  $\varepsilon \notin L \in \mathbf{NSPACE}(n)$  implies that  $L$  is context-sensitive.

### Problem 4 (10 Points)

Prove that the following language is **PSPACE**-complete:

IN-PLACE ACCEPTANCE: Given a Turing machine  $M$  and an input  $x$ , does  $M$  accept  $x$  without ever leaving the first  $|x| + 1$  symbols on its strings?