Technische Universität München Fakultät für Informatik Lehrstuhl für Effiziente Algorithmen Prof. Dr. Ernst W. Mayr Chris Pinkau

Complexity Theory

Due date: June 22, 2015 before class!

Problem 1 (10 Points)

Show the details of the proof of Theorem 5.6. from the lecture: For every $i \ge 1$, if $\Sigma_i^p = \Pi_i^p$, then $\mathbf{PH} = \Sigma_i^p$.

Problem 2 (10 Points)

Recall the definition of alternating Turing machines (ATM) with control states partitioned into sets Q_{\forall} and Q_{\exists} , and the corresponding class **AP**.

- 1. Show that a language $L \in \mathbf{AP}$ decided by an *existential* ATM (i.e. $Q_{\forall} = \emptyset$) is in \mathcal{NP} .
- 2. Show that a language $L \in \mathbf{AP}$ decided by an *universal* ATM (i.e. $Q_{\exists} = \emptyset$) is in $co\mathcal{NP}$.
- 3. Show that $\mathbf{AP} = \mathbf{coAP}$.
- 4. Show that **PSPACE** is contained in **AP** by showing that $TQBF \in AP$.

Problem 3 (10 Points)

Prove $\mathbf{AL} = \mathcal{P}$.