Technische Universität München
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Wintersemester 2015-16
Assignment 12
January 18, 2016

Efficient Algorithms and Datastructures I

Question 1 (10 Points)

The edge connectivity of an undirected graph is the minimum number k of edges that must be removed to disconnect the graph. For example, the edge connectivity of a tree is 1, and the edge connectivity of a cyclic chain of vertices is 2. Show how the edge connectivity of an undirected graph $G = (V, E)$ can be determined by running a maximum-flow algorithm on at most $|V|$ flow networks, each having $O(V)$ vertices and $O(E)$ edges.

Question 2 (10 Points)

A path cover of a directed graph $G = (V, E)$ is a set P of vertex-disjoint paths such that every vertex in V is included in exactly one path in P. Paths may start and end anywhere, and they may be of any length, including 0. A minimum path cover of G is a path cover containing the fewest possible paths.

a Give an efficient algorithm to find a minimum path cover of a directed acyclic graph $G = (V, E)$. (*Hint:* Assuming that $V = \{1, 2, \dots, n\}$, construct the graph $G' = (V', E')$, where

$$\begin{aligned} V' &= \{x_0, x_1, \dots, x_n\} \cup \{y_0, y_1, \dots, y_n\}, \\ E' &= \{(x_0, x_i) : i \in V\} \cup \{(y_i, y_0) : i \in V\} \cup \{(x_i, y_j) : (i, j) \in E\} \end{aligned}$$

and run a maximum-flow algorithm.)

b Does your algorithm work for directed graphs that contain cycles? Explain.

Question 3 (10 Points)

We say that a bipartite graph $G = (V, E)$, where $V = L \cup R$, is d-regular if every vertex $v \in V$ has degree exactly d. Every d-regular bipartite graph has $|L| = |R|$. Prove that every d-regular bipartite graph has a matching of cardinality $|L|$ by arguing that a minimum cut of the corresponding flow network has capacity $|L|$.

Question 4 (10 Points)

A town has r residents $R_1, \dots, R_r$, q clubs $C_1, \dots, C_q$, and p political parties $P_1, \dots, P_p$. Each resident is a member of at least one club and belongs to exactly one political party. Each club must nominate one of its members to represent it on the town's governing council so that the number of council members belonging to the political party P_k is at most u_k. Using maxflows, find out whether it is possible for clubs nominate members in such a way.