OVERLAY NETWORKS FOR WIRELESS AD HOC NETWORKS

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Abstract. Radio networks are widely used today. People access voice and data services via mobile phones, Bluetooth technology replaces unhandy cables by wireless links, and wireless networking is possible via IEEE 802.11 compatible network equipment. Nodes in such networks exchange their data packets usually with fixed base stations that connect them with a wired backbone. However, in applications such as search and rescue missions or environmental monitoring, no explicit communication infrastructure may be available. In this case, the wireless hosts have to organize in a so-called wireless ad hoc network. As long as all of the hosts are within transmission range of each other, the problem of exchanging information in such a network basically boils down to designing suitable medium access control protocols, but if not all hosts can directly communicate with each other, we also need suitable routing algorithms. Designing routing algorithms for wireless ad hoc networks is an extremely challenging task and still research in progress. In this paper, we mostly focus on the simpler question of how to maintain an overlay network of wireless links between the hosts so that, as a minimum requirement, every node is reachable from every other node (i.e. the graph formed by the links is connected) as long as this is possible. Ideally, for every pair of nodes (v, w) there should also be a route from v to w with a close to minimum possible hop distance or energy consumption. The graph formed by the wireless links should also have a low degree to ensure a low maintenance cost and it should be easy to update in case of arrivals or departures of nodes or changes in their positions. This paper will present various strategies for reaching these goals under ideal as well as (more) realistic models.

Key words. Wireless ad hoc networks, overlay networks, spanner, wireless models.

AMS(MOS) subject classifications. 68M10, 68R10, 90B18

1. Introduction.

The problem of designing an overlay network for wireless ad hoc networks has recently attracted a lot of attention. A basic requirement for these overlay network designs is that they maintain connectivity among the hosts, as long as this is possible. The most straightforward approach to achieve connectivity is to maintain a link between every pair of wireless hosts that are within their transmission range. However, this may require a high maintenance and update cost since the corresponding overlay network may have a high degree. Also, some links may have a high energy cost, and so a natural question would be whether these can be dropped without endangering connectivity.

An alternative approach would be to maintain connections only to the k nearest neighbors. However, Figure 1 demonstrates that it is easy to come up with examples in which the graph formed by the links would not

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be connected. So this approach does not work in general. As was shown by Xue and Kumar [30], it only works in specific cases. For example, if n hosts are distributed uniformly at random in a unit square and every host connects to more than $5.1774 \log n$ of its nearest neighbors, then the network formed by these links is connected with a probability that tends to 1 as n increases. But connecting to less than $0.074 \log n$ nearest neighbors results in almost sure disconnectivity.

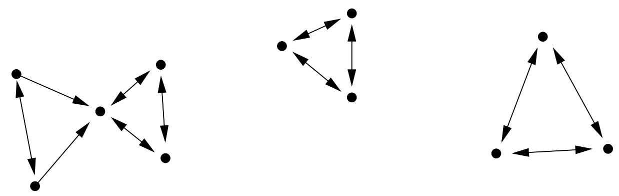

FIG. 1. *A counterexample for the naive approach with $k = 2$.*

Another possible approach is that every host maintains connections to k hosts chosen uniformly at random among all hosts within its transmission range. This also does not guarantee connectivity in general but works well in certain cases. For example, Dubhashi et al. [8] recently showed that if every node has at least $\Theta(\log n)$ nodes within its transmission range, then choosing just 2 random nodes to connect to will establish connectivity almost surely.

In this paper, we are only focusing on approaches that *guarantee* connectivity no matter how the hosts are distributed, as long as this is in principle possible. Most of these approaches are based on so-called *spanners*, which are properly selected subgraphs of the graph of all possible connections between the wireless hosts so that the hosts are not only connected but their (hop or Euclidean) distance in that graph is closely related to their minimum (hop or Euclidean) distance when considering all possible connections. Spanners first appeared in computational geometry [10, 31], were then discovered as an interesting tool for approximating NP-hard problems [24], and have recently attracted a lot of attention in the context of routing and topology control in wireless ad hoc networks [1, 11, 12, 3, 23].

In the following, the wireless hosts are simply called *nodes*. To simplify our presentation, we assume that the nodes are distributed in a perfect 2-dimensional Euclidean space, or formally, the nodes represent a set of points $V \subset \mathbb{R}^2$, but all of the approaches presented here can also be extended to higher dimensions. Given any pair of nodes $u = (u_x, u_y), v = (v_x, v_y) \in \mathbb{R}^2$,

$$||uv|| = \sqrt{(u_x - v_x)^2 + (u_y - v_y)^2}$$

denotes the *Euclidean distance* between u and v, and given any sequence of nodes $s = (u_1, u_2, \ldots, u_k)$ and any $\delta \geq 0$,

$$||s||^\delta = \sum_{i=1}^{k-1} ||u_i u_{i+1}||^\delta$$

denotes the δ-*cost* of s. For any graph $G = (V, E)$, a node sequence $s = (u_1, u_2, \ldots, u_k)$ is called a *path* in G if $(u_i, u_{i+1}) \in E$ for all $1 \leq i < k$.

Given any directed graph $G = (V, E)$ and any two nodes $u, v \in V$, the δ-*distance* $d_G^\delta(u, v)$ of u and v in G is the minimum δ-cost $||p||^\delta$ over all paths p from u to v in G. If $\delta = 0$, then $d_G^\delta(u, v)$ gives the *topological (or hop) distance* of u and v in G, and if $\delta = 1$, $d_G^\delta(u, v)$ gives the *Euclidean distance* of u and v in G. Also cases with $\delta > 1$ are interesting for us because the transmission of a packet over a distance of r usually has an energy consumption that scales with r^δ for some $\delta > 1$. In reality, δ is usually in the range $[2, 5]$, where it is closer to 2 outdoors and closer to 5 indoors.

We assume that every node has a maximum transmission range of 1, i.e., every node $u \in V$ can send messages only to nodes $v \in V$ with $||uv|| \leq 1$. From this assumption it follows that every overlay network connecting these nodes can only be a subgraph of the following graph.

DEFINITION 1.1. *For any point set* $V \subset \mathbb{R}^2$*, the* unit disk graph *of* V*, called* $UDG(V)$*, is a directed graph that contains all edges* (u, v) *with* $||uv|| \leq 1$.

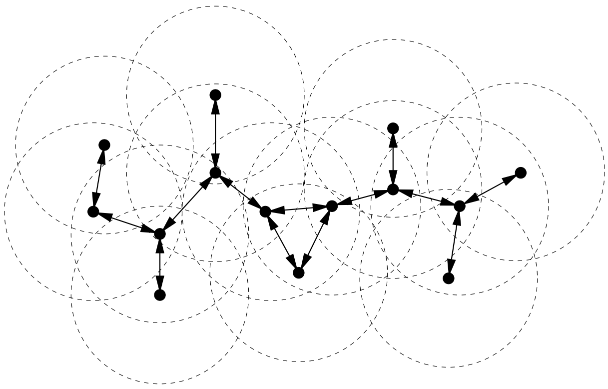

FIG. 2. *A connected unit disk graph.*

In the following, we will always assume that V is chosen so that its UDG is connected and non-degenerate, i.e., there is a path in $UDG(V)$ between every pair of nodes and no two pairs of nodes have exactly the same Euclidean distance (see also Figure 2). The connectivity assumption is a prerequisite for our strategies below to establish a connected network among the nodes and the non-degenerateness property will simplify the proofs. When G is the UDG of V, we simply use $d^\delta(u, v)$ instead of $d_G^\delta(u, v)$.

1.1. Structure of the paper. The rest of this paper is organized as follows. First, we define different kinds of geometric spanners and provide relationships between them (Section 2.1). Afterwards, we study a general class of graphs called *proximity graphs* that contain many of the spanner constructions proposed for ad hoc networks (Section 2.2). Among these spanner constructions are sector-based spanners and planar spanners. Various sector-based spanners are reviewed in Section 2.3, and various planar spanners are reviewed in Section 2.4. All of these constructions are based on simple space and energy models. Namely, the nodes are distributed in a perfect 2-dimensional Euclidean space, every node has a transmission radius of 1 and the energy necessary for transmitting a message over a distance of d is d^δ for some fixed constant $\delta \geq 2$. In Section 3 we show how to modify the spanner constructions in Section 2 so that even under more realistic models the spanner constructions still work. The paper ends with conclusions.

2. Spanners. First, we define spanners in which arbitrary pairs of nodes can, in principle, be connected by an edge (i.e., we do not limit the transmission range of nodes).

DEFINITION 2.1. *Consider any finite set of nodes $V \subset \mathbb{R}^2$, and let $c \geq 1$ be any constant.*

- *A graph $G = (V, E)$ is called a* geometric c-spanner *of V if for all $u, v \in V$ there exists a path p from u to v in G with*

$$||p|| \leq c \cdot ||uv|| \ .$$

  If G is a geometric c-spanner, c is called its stretch factor.
- *G is a (c, δ)*-power spanner *of V if for all $u, v \in V$ there is a path p from u to v in G with*

$$||p||^\delta \leq c \cdot ||uv||^\delta \ .$$

  If for all $\delta \geq 2$ there exists a constant c so that G is a (c, δ)-power spanner, then we simply call G a power spanner.
- *G is a* weak c-spanner *of V if for all $u, v \in V$ there is a path p from u to v in G that is within a disk of diameter at most*

$$c \cdot ||uv|| \ .$$

- *A graph $G = (V, E)$ is called a* constrained *(geometric, power, or weak) spanner of V if for every pair of nodes $u, v \in V$ there is a path p that, in addition to the specific requirement for the spanner type, also satisfies the condition that for every edge e in p,*

$$||e|| \leq ||uv|| \ .$$

Since wireless nodes have a limited transmission range, the following spanner definitions are more relevant for ad hoc networks.

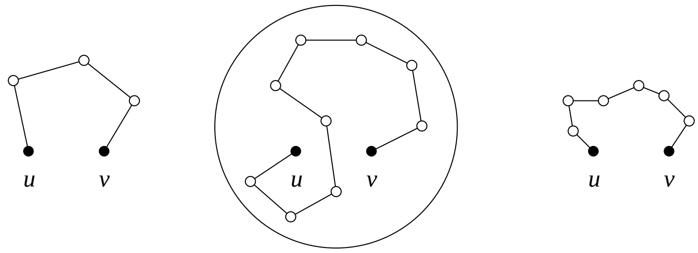

FIG. 3. *Examples of a spanner, weak spanner, and power spanner.*

DEFINITION 2.2. *Let $V \subset \mathbb{R}^2$ be any finite set of nodes with a connected UDG.*

- *A graph $G = (V, E)$ is called a* geometric c-spanner *of $UDG(V)$ if for all $u, v \in V$ there exists a path p from u to v in G with*

$$||p|| \leq c \cdot d(u, v) \ .$$

- *G is a* (c, δ)-power spanner *of $UDG(V)$ if for all $u, v \in V$ there is a path p from u to v in G with*

$$||p||^\delta \leq c \cdot d^\delta(u, v) \ .$$

- *G is a* weak c-spanner *of $UDG(V)$ if for all $u, v \in V$ there is a path p from u to v in G that is within a disk of diameter at most*

$$c \cdot d(u, v) \ .$$

Interestingly, any constrained spanner of V in which all edges of length more than 1 are removed is also a spanner of the UDG of V, as shown in the next theorem.

THEOREM 2.1. *Any constrained geometric c-spanner / (c, δ)-power spanner / weak c-spanner G of V restricted to edges of length at most 1 is also a geometric c-spanner / (c, δ)-power spanner / weak c-spanner of the UDG of V.*

Proof. Let U be the UDG of V. Suppose that G is a (c, δ)-power spanner of V for some $\delta \geq 0$. Then it holds for every pair of nodes $u, v \in V$ with $||uv|| \leq 1$ that there is a path p in $G \cap U$ with $||p||^\delta \leq c||uv||^\delta$. Now, consider an arbitrary pair $u, w \in V$, and let $p = (v_0, v_1, v_2, \ldots, v_k)$ be any path in U with $v_0 = u$ and $v_k = w$ that has a δ-cost of $d^\delta(u, w)$. Since $||v_i v_{i+1}|| \leq 1$ for all i, there is a path p_i from v_i to v_{i+1} in $G \cap U$ with $||p_i||^\delta \leq c||v_i v_{i+1}||^\delta$. Concatenating these paths, we end up with a path p' with

$$||p'||^\delta = \sum_{i=0}^{k-1} ||p_i||^\delta \leq \sum_{i=0}^{k-1} c||v_i v_{i+1}||^\delta = c \cdot d^\delta(u, w) \ .$$

Hence, $G \cap U$ is also a (c, δ)-power spanner of U. Since a geometric c-spanner is just a $(c, 1)$-power spanner, this also proves the theorem for constrained geometric spanners.

Finally, consider the case that G is a constrained weak c-spanner. Then it holds for every pair of nodes $u, v \in V$ with $||uv|| \leq 1$ that there is a path p in $G \cap U$ that is within a disk of diameter at most $c||uv||$. Consider now an arbitrary pair $u, w \in V$, and let $p = (v_0, v_1, v_2, \ldots, v_k)$ be any path in U with $v_0 = u$ and $v_k = w$ that has a Euclidean length of $d(u, w)$. Since $||v_i v_{i+1}|| \leq 1$ for all i, there is a path p_i from v_i to v_{i+1} in $G \cap U$ that is within a disk of diameter at most $c \cdot ||v_i v_{i+1}||$. Concatenating these paths, we end up with a path p' that is within a disk of diameter at most $c \cdot d(u, w)$. To prove this, we need the following straightforward fact.

FACT 2.2. *Any two disks of diameter d_1 and d_2 with a non-empty intersection are contained in a disk of diameter at most $d_1 + d_2$.*

Using this fact in an inductive manner on the length of p, it follows that when replacing the paths p_i in p' by their disks, p' is contained in a disk of radius at most

$$\sum_{i=0}^{k-1} c \cdot ||v_i v_{i+1}|| \leq c \cdot d(u, w) \ .$$

□

Hence, it suffices to present and analyze algorithms for constrained spanners in order to obtain overlay networks that are also spanners of UDGs.

2.1. Relationships between spanners. Next, we study general relationships between the different kinds of spanners. All of these relationships hold for general spanners as well as constrained spanners. To keep the paper at a reasonable size, most of the proofs are left out. For detailed proofs, see [26, 28]. We start with a straight-forward theorem.

THEOREM 2.3. *Every graph $G = (V, E)$ that is a (constrained) geometric c-spanner is also a (constrained) weak c-spanner.*

However, the theorem does not hold any more when considering power spanners.

THEOREM 2.4 ([28]). *For any $\delta > 1$ there is a family of (constrained) (c, δ)-power spanners which are not a (constrained) weak C-spanner for any constant C.*

Also, the reverse direction of Theorem 2.3 is not true, i.e., the fact that a graph is a weak spanner does not imply in general that it is also a geometric spanner.

THEOREM 2.5 ([28]). *There exists a family of graphs $G = (V, E)$ with $V \subset \mathbb{R}^2$ all of which are (constrained) weak $2(\sqrt{2}+1)$-spanners but not a (constrained) geometric c-spanner for any constant c.*

The next theorem studies the relationship between geometric spanners and power spanners, which is easy to show.

THEOREM 2.6. *Every (constrained) geometric c-spanner is a (constrained) (c^δ, δ)-power spanner for every $\delta \geq 1$.*

Hence, in order to prove that a graph is a power spanner, it suffices to prove that it is a geometric spanner. Interestingly, for $\delta \geq 2$, it even suffices to show that a graph is a weak spanner in order to prove that it is a power spanner.

THEOREM 2.7 ([28]). *Let $G = (V, E)$ be a (constrained) weak c-spanner. Then G is also a (constrained) (C, δ)-power spanner for $\delta > 2$ where $C = (4c + 1)^2 \cdot \frac{c^\delta}{1-2^{2-\delta}}$. It is even a weak spanner for $\delta = 2$.*

However, a weak c-spanner may not be a (C, δ)-power spanner for any constant C if $\delta < 2$.

THEOREM 2.8 ([28]). *For any $\delta < 2$ there exists a family of graphs $G = (V, E)$ with $V \subset \mathbb{R}^2$ which are (constrained) weak c-spanners for a constant c but not a (constrained) (C, δ)-power spanner for any constant C.*

Summing up Theorems 2.3, 2.4, 2.5, 2.6, and 2.7, we obtain the following interesting relationship between the class of all geometric spanners, weak spanners, and power spanners with $\delta \geq 2$:

Geometric spanners $\subset$ Weak spanners $\subset$ Power spanners

2.2. Proximity graphs. From our insights on spanners above it follows that it would often be sufficient to design protocols that guarantee a constrained weak c-spanner as long as this is possible because weak spanners are guaranteed to have energy-efficient paths. But how can such spanners be designed? Consider the following definition:

DEFINITION 2.3. *For any node set $V \subset \mathbb{R}^2$, the graph $G = (V, E)$ is called a* proximity graph *of V if and only if for all $u, w \in V$ it holds that*

- *$(u, w) \in E$ or*
- *there is a $v \in V$ with $(u, v) \in E$ and $||vw|| < ||uw||$.*

For an example of a node v satisfying the proximity conditions, see Figure 4. It is known that there are proximity graphs with a stretch factor as bad as $|V| - 1$ [4] but proximity graphs are always good weak spanners.

THEOREM 2.9. *For any finite $V \subset \mathbb{R}^2$, every proximity graph of V is a weak 2-spanner.*

Proof. Let $G = (V, E)$ be any proximity graph of V. First we prove that G is connected. Certainly, a graph G is connected if and only if for every pair of nodes in G there is a path connecting these two nodes. So consider any pair of nodes $u, w \in V$. We distinguish between two cases:

1. $(u, w) \in E$: Then u and w are connected, and we are done.
2. There is a $v \in V$ with $(u, v) \in E$ and $||vw|| < ||uw||$: Then we use the edge (u, v) and get closer to w then we were before.

Since V is finite, we only have to apply case 2 a finite number of times until case 1 holds. Hence, G is connected.

Besides G being connected, it follows from the observation above that for any pair of nodes $u, w \in V$ there is a path p that monotonically converges against w. Hence, p is contained in a disk of diameter at most $2||uw||$, which proves the theorem. □

Hence, every proximity graph is also a power spanner of V for every $\delta \geq 2$. To make proximity graphs useful for ad hoc networks, we consider a constrained form of proximity graphs which are also known as relative neighborhood graphs [4].

DEFINITION 2.4. *For any node set $V \subset \mathbb{R}^2$, the graph $G = (V, E)$ is called a* relative neighborhood graph *(RNG) of V if and only if for all $u, w \in V$ it holds that*

- *$(u, w) \in E$ or*
- *there is a $v \in V$ with $(u, v) \in E$, $||uv|| < ||uw||$, and $||vw|| < ||uw||$.*

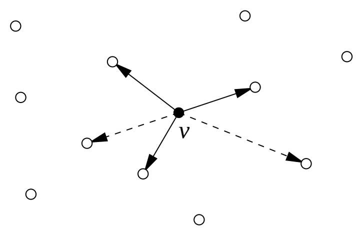

FIG. 4. *Connections satisfying the RNG condition for v. (Removing the dashed connections gives a minimum set of connections satisfying the RNG condition.)*

It is easy to verify that relative neighborhood graphs satisfy the condition on constrained graphs we formulated for spanners in Definition 2.2. Hence, Theorems 2.1, 2.7, and 2.9 imply that relative neighborhood graphs are weak and power spanners of the UDG of V for every $\delta \geq 2$.

Though relative neighborhood graphs may be good weak spanners, they may not be geometric spanners or power spanners with a low cost. Here, two basic approaches have been pursued in the literature to obtain geometric spanners and/or power spanners with low cost:

- The nodes cut the space around them into sectors of equal angle θ, where θ is sufficiently small. Such graphs are also known as θ-graphs or Yao graphs [6, 25, 31].
- The nodes triangulate the space to form Delaunay-like graphs.

We first consider Yao graphs and their variants, which we also call *sector-based spanners*, and afterwards we study Delaunay graphs and their variants, which we also call *planar spanners*.

2.3. Sector-based spanners. The basic idea underlying the Yao graphs is to cut the space around each node into sectors of equal angle θ and to connect each node to the nearest neighbor in each of its sectors (see Figure 5). As we will see, this will give a relative neighborhood graph if θ is sufficiently small. For any pair of nodes u, v, let $C_{u,v}$ denote the sector (or cone) of u containing v.

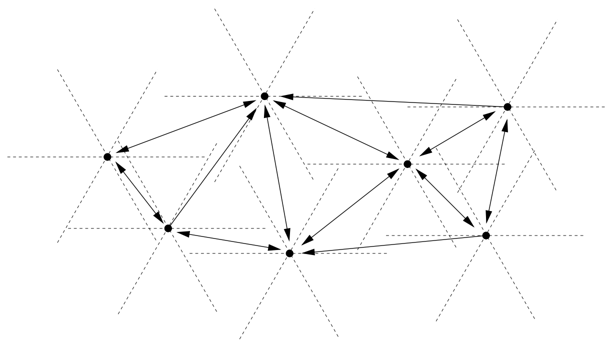

FIG. 5. *An example of a Yao graph.*

DEFINITION 2.5. *Consider any finite $V \subset \mathbb{R}^2$ and let $k \in \mathbb{N}$. Suppose that the space around every node $v \in V$ is cut into k sectors with angle $\theta = 2\pi/k$. Then the* Yao graph *$YG_\theta(V)$ of V consists of the following set of edges:*

$$E = \{(u, v) \mid u, v \in V \text{ and there is no } w \in V \text{ with } w \in C_{u,v} \text{ and } ||uw|| < ||uv||\} .$$

We start with a basic property of Yao graphs.

THEOREM 2.10. *If $\theta = 2\pi/k$ with $k > 6$, then $YG_\theta(V)$ is a RNG.*

The theorem immediately implies that Yao graphs with $k > 6$ are weak spanners. But they are more than that, as shown in the next theorem.

THEOREM 2.11 ([25]). *If $\theta = 2\pi/k$ with $k > 6$, then $YG_\theta(V)$ is a geometric spanner with stretch factor at most*

$$\frac{1}{1 - 2\sin(\theta/2)} .$$

Combining this with Theorem 2.6 yields the following result.

COROLLARY 2.1. *If $\theta = 2\pi/k$ with $k > 6$, then $YG_\theta(V)$ is a (c, δ)-power spanner for every $\delta \geq 1$ with*

$$c \leq \left(\frac{1}{1 - 2\sin(\theta/2)}\right)^\delta .$$

A much better result was shown by Li et al. [21] for $\delta \geq 2$. We recently strengthened their result to any $\delta \geq 1$.

THEOREM 2.12 ([26]). *If $\theta = 2\pi/k$ with $k > 6$, then $YG_\theta(V)$ is a (c, δ)-power spanner for every $\delta \geq 1$ with*

$$c \leq \frac{1}{1 - (2\sin(\theta/2))^\delta} .$$

The drawback of the Yao graph is that, although its out-degree is at most k, its in-degree may be as high as $n-1$ (consider, for example, the disk in Figure 9 with one node in its center and all other nodes on its border). Various sub-graphs of the Yao graph have been suggested to remove this drawback. We will present two of them here (see also [3]).

DEFINITION 2.6. *The* sparsified Yao graph $SpYG_\theta(V)$ *is a sub-graph of* $YG_\theta(V)$ *with edge set*

$$E = \{(u,v) \in E(YG_\theta(V)) \mid \textit{for all } w \in V \textit{ with } (w,v) \in E(YG_\theta(V)) \textit{ and } w \in C_{v,u} : \ ||vw|| > ||vu||\} .$$

In words, for every sector of every node v, the sparsified Yao graph only keeps the shortest of all edges into v. Hence, the sparsified Yao graph has an in-degree of at most k and an outdegree of at most k, and therefore a degree of at most $2k$.

DEFINITION 2.7. *The* symmetric Yao graph $SyYG_\theta(V)$ *is a sub-graph of* $YG_\theta(V)$ *with edge set*

$$E = \{(u,v) \in E(YG_\theta(V)) \mid (v,u) \in E(YG_\theta(V))\} .$$

In words, the symmetric Yao graph only keeps an edge (u,v) if not only v is the nearest neighbor of u in $C_{u,v}$ but also u is the nearest neighbor of v in $C_{v,u}$. Hence, the symmetric Yao graph has a degree of at most k. Obviously,

$$SyYG_\theta(V) \subseteq SpYG_\theta(V) \subseteq YG_\theta(V)$$

and Figure 6 shows that there are cases in which the edge sets of the different graphs are proper subsets of each other. Thus, it suffices to prove connectivity for $SyYG_\theta(V)$ in order to prove connectivity for both variants of the Yao graph.

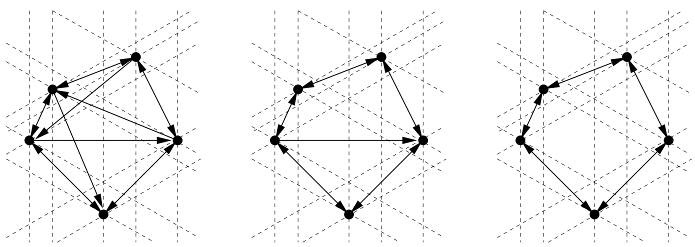

FIG. 6. *The Yao graph, the sparsified Yao graph, and the symmetric Yao graph of a point set.*

THEOREM 2.13 ([11]). *For all non-degenerate node sets V and $k > 6$, $SyYG_\theta(V)$ is connected.*

Unfortunately, the symmetric Yao graph is not a good power spanner for any $\delta \geq 1$, which implies that it is not even a good weak spanner.

THEOREM 2.14 ([3]). *The symmetric Yao graph is not a (c, δ)-power spanner for any constant c and any $\delta \geq 1$.*

However, the sparsified Yao graph is a good weak spanner.

THEOREM 2.15 ([3]). *If $k > 6$, then the sparsified Yao graph is a weak c-spanner with $c = \frac{2}{1-2\sin(\theta/2)}$.*

Though the sparsified Yao graph is not a relative neighborhood graph like the original Yao graph, it is easy to check that when restricting to the UDG of V, the proof of Theorem 2.15 is still correct for all pairs u, w with $||uw|| \leq 1$. Hence, it follows from the proof of Theorem 2.1 that the sparsified Yao graph is also a weak c-spanner of the UDG of V. Thus, Theorem 2.7 implies that it is also a power spanner of the UDG of V for every $\delta \geq 2$ and therefore useful for wireless ad hoc networks.

2.4. Planar spanners. The most well known class of planar spanners are the Delaunay graphs. The Delaunay graph of a set of points in $\mathbb{R}^2$ is equivalent to their Delaunay triangulation and the dual of their Voronoi diagram. Since the Delaunay triangulation of any point set in $\mathbb{R}^2$ is planar, the Delaunay graph is planar. In the following, let $\triangle(uvw)$ be the triangle formed by the nodes u, v, and w and $\bigcirc(uvw)$ be the unique circle through u, v, and w.

DEFINITION 2.8. *For any $V \subset \mathbb{R}^2$, the* Delaunay graph $Del(V)$ *of V consists of all edges (u, v) that have a node $w \in V$ for which $\bigcirc(uvw)$ does not contain any other node of V.*

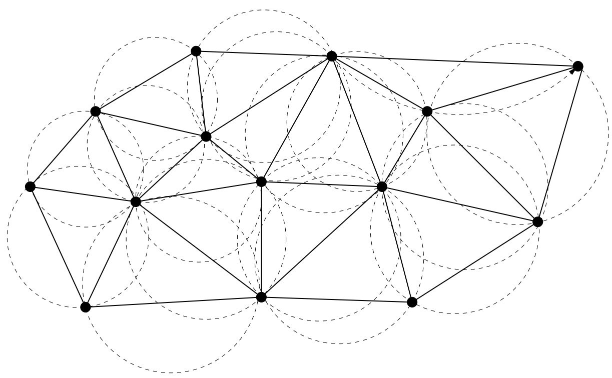

FIG. 7. *An example of a Delaunay graph.*

For an example of a Delaunay graph see Figure 7. It is known [7, 14] that the Delauney graph is a geometric c spanner with $c = \frac{2\pi}{3\cos(\pi/6)} \approx 2.42$, but the Delaunay graph is difficult to maintain locally. Therefore, several variants of it have been proposed. The most well-known variant is the Gabriel graph.

DEFINITION 2.9. *For any* $V \subset \mathbb{R}^2$, *the* Gabriel graph $GG(V)$ *of* V *consists of all edges* (u, w) *with the property that there is no node* $v \in V$ *with*

$$||uv||^2 + ||vw||^2 < ||uw||^2 .$$

In words, the Gabriel graph of V consists of all edges $\{u, w\}$ with the property that the open sphere through u and w with diameter $||uw||$ does not contain any other node in V. An example of a Gabriel graph is given in Figure 8. The Gabriel graph has the following interesting properties:

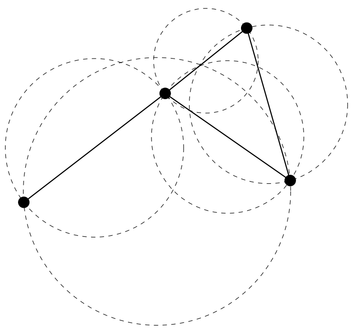

FIG. 8. *A Gabriel graph.*

THEOREM 2.16. *For any* $V \subset \mathbb{R}^2$, *the Gabriel graph of* V *is a relative neighborhood graph and a subgraph of the Delaunay graph of* V.

Unfortunately, Theorem 2.5 implies that the Gabriel graph is not a geometric spanner. With better techniques one can even create a counterexample with stretch factor $\Omega(\sqrt{n})$ [21]. But Theorem 2.9 implies that the Gabriel graph is a weak 2-spanner, and even more importantly, it is an optimal power spanner for every $\delta \geq 2$.

THEOREM 2.17 ([21]). *For every* $\delta \geq 2$, *the Gabriel graph is an optimal power spanner.*

Unfortunately, the outdegree of a Gabriel graph can be as high as $n-1$ (see Figure 9). Also, since the Gabriel graph is not a geometric spanner, one may ask whether there are locally constructible planar graphs that are geometric spanners. To investigate the latter issue, we define the following classes of graphs.

DEFINITION 2.10. *A triangle* $\triangle(uvw)$ *satisfies the* k-localized Delaunay property *if the interior of the disk* $\bigcirc(uvw)$ *does not contain any node of* V *that is a* k*-neighbor of* u, v, *or* w *in* $UDG(V)$ *and* $(u, v), (v, w), (w, u) \in UDG(V)$. *Such a triangle is called a* k-localized Delaunay triangle.

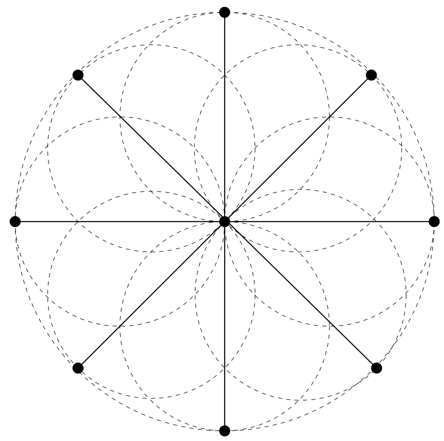

FIG. 9. *Gabriel graph for the unit disk with one node in its center and all other nodes on its border.*

DEFINITION 2.11. *The* k-localized Delaunay graph *over* V*, denoted by* $LDel^{(k)}(V)$*, has exactly all Gabriel edges and the edges of all* k*-localized Delaunay triangles.*

Let the *constrained Delaunay graph* of a point set V be defined as $UDel(V) = Del(V) \cap UDG(V)$. The following facts are known about k-localized Delaunay graphs.

THEOREM 2.18 ([20]). *Localized Delaunay graphs have the following properties:*

1. $UDel(V) \subseteq LDel^{(k)}(V)$ *for all* $k \geq 1$.
2. $LDel^{(k+1)}(V) \subseteq LDel^{(k)}(V)$ *for all* $k \geq 1$.
3. $LDel^{(2)}(V)$ *is a planar graph.*
4. $LDel^{(1)}(V)$ *is not always planar.*

Since $UDel(V)$ is a geometric c-spanner with $c \approx 2.42$ it follows that $LDel^{(2)}(V)$ is a geometric c-spanner with $c \approx 2.42$, and it is also planar.

However, as mentioned above, Gabriel graphs and therefore all graphs of the localized Delaunay graph family have the problem that the degree may be very high (see Figure 9). This problem can be solved by constraining a Delaunay graph in the same way Yao graphs are constrained to sparsified Yao graphs: cut the space around each node into $k > 6$ sectors of equal angle, and accept only the connection of the closest node with an incoming edge in the original graph. Similar to the proof of the sparsified Yao graph, this gives a sparsified Delaunay graph that is still a weak spanner. Other constructions have been proposed that can even maintain a Euclidean $O(1)$-spanner but at the cost of requiring an algorithm that may need a long time to stabilize at some solution [29].

3. From ideal to realistic models. In the previous section we saw that in the ideal world (i.e., a perfect 2-dimensional Euclidean space) it is possible to construct good spanners. But how about the real world? In the real world, many assumptions made above are not valid any more. For example, instead of the unit disk graph, more involved models have to be chosen to model the transmission range of a node in real life. The position of a node or its distance or angle to another node may not be easy to determine. The energy consumed by transmitting a message over a distance of d is not simply d^δ for some fixed $\delta \geq 2$. Also, the spanner constructions above can create very dense networks that can possibly create a lot of contention, reducing the effectiveness of wireless communication in practice. Finally, mobility has not been addressed above. We will present possible solutions to each of these problems.

3.1. Unit disk model. Certainly, the unit disk model is too simplistic to model the transmission range of wireless nodes. One alternative model would be the standard packet radio network model used in many papers on wireless broadcasting:

We model the wireless medium as a graph $G = (V, E)$ where V represents the set of wireless nodes and $(u, v) \in E$ if and only if u is able to transmit a message to v.

This model has two disadvantages. First of all, it is too general. Pathological cases can be constructed that would never occur in practice. It is possible, for example, to choose a graph G that makes it impossible to construct a low-degree (and therefore low contention) geometric spanner. In fact, it is easy to come up with a node distribution V and graph G where any geometric spanner with constant stretch factor would have to be a star graph, i.e., one node in it must have a degree of $n - 1$. Also, the packet radio network model does not allow us to say how the transmission range of a node changes when changing its transmission power. An alternative model could be the following:

We are given a set V of wireless nodes that are distributed in an arbitrary way in a 2-dimensional Euclidean space. Consider any function t with the property that there is a fixed constant $\gamma \in [0, 1)$ so that for any two points p and q in the Euclidean space,

1. $t(p, q) \in [(1 - \gamma) \cdot ||pq||,\ (1 + \gamma) \cdot ||pq||]$ and
2. $t(p, q) = t(q, p)$, i.e. t is symmetric.

For any two nodes $u, v \in V$ where u sends with transmission power corresponding to a value of t_u, v is in the transmission range of u if and only if $t(u, v) \leq t_u$. Applying the UDG model to this new model, this means that two nodes u and v are within transmission range if $t(u, v) \leq 1$. Thus, t determines the transmission range of the nodes and γ bounds the non-uniformity of the environment. Notice that we do not require t to be monotonic in the distance or to satisfy the triangle inequality. This makes sure that our model even applies to highly irregular environments. In Figure 10, for example, the distance between u and v is greater than the distance between u and w. Yet, the cost of communicating between u and

w, $t(u,w)$, is bigger than $t(u,v)$. Similar cost functions were also used in [18].

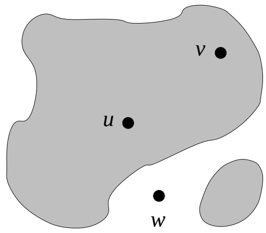

FIG. 10. *The area covered by the maximum transmission range of node u is given by the shaded area. Given a maximum transmission range of 1, this means that* $t(u,v) \leq 1$ *and* $t(u,w) > 1$.

Does the cost model still allow us to construct good spanners? Yes, it does, because of condition 1 on the cost function above. This condition essentially states that $t(p,q)^\delta = \Theta(||pq||^\delta)$ for any constant $\delta \geq 0$. Hence, the following fact holds.

FACT 3.1. *For any graph G it holds that G is a geometric c-spanner / (c,δ)-power spanner / weak c-spanner of V w.r.t. $||\cdot||$ if and only if G is a geometric c-spanner / (c,δ)-power spanner / weak c-spanner of V w.r.t. t.*

Using this fact, it is easy to verify that Theorem 2.1 still holds. Also, the relationships between the different classes of spanners in Section 2.1 still hold due to Fact 3.1. For the various spanner constructions that we suggested afterwards, we obtain the following results.

Proximity graphs. Definition 2.3 with $||\cdot||$ being replaced by t still satisfies Theorem 2.9 since for any pair of nodes $u,w \in V$ in a proximity graph $G=(V,E)$, either $(u,w) \in E$ or $(u,v) \in E$ for some node v with $t(v,w) < t(u,w)$. Hence, all nodes on a path from u to w lie within a transmission range of at most $t(u,w)$ around w.

Sector-based spanners. To make sure that the Yao graph is still a geometric spanner, an angle θ has to be chosen so that for any two nodes $u,w \in V$ it holds that either $(u,w) \in E$ or there is a node v in the sector $C_{u,w}$ with $(u,v) \in E$ and

$$t(v,w) \leq (1-\epsilon)t(u,w)$$

for some constant $\epsilon > 0$. For $\gamma = 0$ in the conditions for t this is true for any $\theta < 2\pi/k$. If $\gamma > 0$, then it can be shown via trigonometric arguments (see also Figure 11) that

$$t(v,w) \leq (1+\gamma)\sqrt{\left(\frac{t(u,v)\sin\theta}{1+\gamma}\right)^2 + \left(\frac{t(u,v)}{1-\gamma} - \left(\frac{t(u,v)\cos\theta}{1+\gamma}\right)\right)^2}\,.$$

Simplifying this expression, we obtain

$$t(v,w) \le \frac{\sqrt{2(1-\cos\theta)+2\gamma^2(1+\cos\theta)}}{1-\gamma} \cdot t(u,v) \ .$$

It holds that

$$\frac{\sqrt{2(1-\cos\theta)+2\gamma^2(1+\cos\theta)}}{1-\gamma} < 1 \quad \Leftrightarrow \quad \cos\theta > \frac{1}{2(1-\gamma)} \ .$$

Hence, as long as $\theta \le \arccos(1/(2(1-\gamma))) - \epsilon$ for some constant $\epsilon > 0$, which is only possible if $\gamma < 1/2$, the Yao graph construction still yields a geometric spanner of constant stretch factor. Under this condition on θ, also the properties for the sparsified and the symmetric Yao graph can be shown.

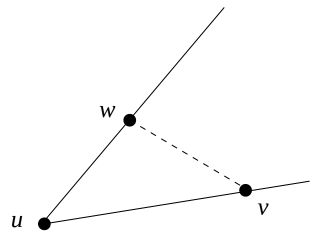

FIG. 11. *Node u has a connection to v because $t(u,v) < t(u,w)$, but w may be closer to u then v if $\gamma > 0$.*

Planar spanners. The planarity condition cannot be satisfied any more because the Delaunay condition of having no node in $\bigcirc(uvw)$ can create crossing edges when applying this condition to t. These crossing edges can be very hard to determine (see, for example, Figure 12). Nevertheless, the Gabriel graph is still a RNG due to the condition that $t(u,v)^2 + t(v,w)^2 < t(u,w)^2$, and therefore $t(u,v) < t(u,w)$ and $t(v,w) < t(u,w)$. Hence, the Gabriel graph (and also the other Delaunay graphs presented in Section 2.4 because they are supergraphs of the Gabriel graph) is still a weak spanner.

3.2. Positions, distances and angles. All spanner graph constructions can be easily done in a distributed way if every node knows its position, which is possible if it has GPS. But if the nodes are not able to determine their positions through means like GPS, then other strategies have to be used. In the following, we list some options for the various spanner constructions.

Proximity graphs. In proximity graphs, only the distance has to be computed between nodes. Here, a reasonable strategy might be to measure the signal strength and from there compute the distance based on an appropriate path loss model. This computation may not be too reliable in a non-uniform environment, but as long as the model above with the cost

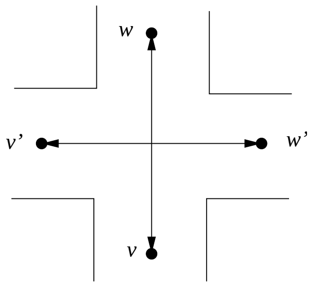

FIG. 12. *The typical street corner problem. Nodes v and w and nodes v' and w' are within transmission range of each other, but v and w cannot reach any one of v' and w', and vice versa.*

function t can be applied (i.e., the path loss only varies by a constant factor), only a constant factor error will be done in the distance calculation. This would suffice to obtain the same properties as shown in the previous subsection.

Sector-based spanners. Here, knowing the distance alone does not suffice because it is also important to compute the angle between the nodes. Here, trilateration techniques may be used: First, u determines the pairwise distance among the nodes in its neighborhood (using the technique mentioned for proximity graphs, for example), and then u tries to lay out the nodes so that all distance relationships are satisfied (up to a small constant factor). Using this virtual layout of the nodes, u will then cut the layout into sectors and connect to nodes according to the Yao graph rules. Again, if the model with the cost function t can be applied, then a similar relationship between θ and the error in the measurement can be shown as in the previous subsection, so that the Yao graph is still a geometric spanner.

Planar spanners. Gabriel graphs do not need to compute the distance between two nodes but may directly use the signal strength. To see this, recall the Gabriel condition

$$||uv||^2 + ||vw||^2 < ||uw||^2 .$$

If the path loss scales quadratically with the distance (which is true in a perfect outdoors environment), then this expression simply states that

$$e(u,v) + e(v,w) < e(u,w)$$

where $e(x,y)$ is the energy necessary to send a message from x to y. But even if this perfect situation is not given, the energy argument still guarantees that the Gabriel graph is a weak spanner as long as we can apply the cost function t above for the path loss. Since the other Delaunay graphs are supergraphs of the Gabriel graph, these will also be weak spanners.

3.3. Energy cost model. So far, we assumed a simple energy cost model, i.e., given a distance of x, the energy consumption scales with $f(x) = x^\delta$ for some $\delta \geq 2$. In reality, there is a fixed minimum energy consumption, so a more realistic function would be $g(x) = \max\{e_0, x^\delta\}$ for some positive constant e_0. In this case, short edges should be avoided because sending a message along many short edges can now be much more expensive than sending a message along a long edge. In fact, in this cost model it is not true any more that every geometric spanner or weak spanner is also a power spanner. So we need to adjust our spanner constructions for this property to be true again. For this we need the concept of a dominating set.

Definition 3.1. *Given a node distribution $V \subset \mathbb{R}^2$ and a distance d, we say that a subset $U \subseteq V$ forms a* dominating set *of V w.r.t. d if for all $v \in V$ either $v \in U$ or there is a node $u \in U$ with $||uv|| \leq d$.*

Suppose now that we found a dominating set U for the given node distribution with respect to distance $d_0 = \sqrt[\delta]{e_0}$. Consider the graph $G = (U, E)$ with E consisting of all edges $\{u, u'\} \in U^2$ with the property that there are $v, v' \in V$ with $||uv|| \leq d_0$, $||u'v'|| \leq d_0$ and $||vv'|| \leq d_{\max}$ where $d_{\max}$ is the maximum transmission range of a node. Since all edges in G have a length of at least d_0, all results about the relationship between geometric, weak and power spanners hold again for G because the threshold e_0 is effectively washed out.

Theorem 3.2. *For any pair $\{u, v\} \in U^2$, the energy of an energy-optimal path $p_{u,v}$ in G is within a constant factor of the energy of an energy-optimal path $p'_{u,v}$ in the $UDG(V)$.*

Proof. Consider any energy-optimal path $p'_{u,v}$ in $UDG(V)$ w.r.t. $g(x)$. Let $v_1, \ldots, v_k$ be the nodes traversed in this path, and let u_i be any node in U with $||u_i v_i|| \leq d_0$ for all $i \in \{1, \ldots, k\}$. Then the energy of the path $p_{u,v} = (v_1, u_1, \ldots, u_k, v_k)$ is within a constant factor of the energy of $p'_{u,v}$ because of the following facts:

- for all i, $g(||v_i v_{i+1}||) = \Omega(e_0)$
- $g(||v_1 u_1||) = O(e_0)$ and $g(||u_k v_k||) = O(e_0)$
- for all i with $||v_i v_{i+1}|| \leq d_0$, $g(||u_i u_{i+1}||) \leq g(3d_0) = (3d_0)^\delta \leq 3^\delta g(||v_i v_{i+1}||) = O(g(||v_i v_{i+1}||))$
- for all i with $||v_i v_{i+1}|| > d_0$, $g(||u_i u_{i+1}||) \leq g(||v_i v_{i+1}|| + 2d_0) \leq (3||v_i v_{i+1}||)^\delta = O(g(||v_i v_{i+1}||))$

□

Hence, in order to repair our spanner constructions for $g(x)$, we can do the following:

1. Choose a dominating set U w.r.t. d_0.
2. Apply the spanner construction to $G = (U, E)$ defined above to obtain a graph $G' = (U, E')$.
3. Construct a graph $G'' = (V, E'')$ out of G' by replacing every edge $\{u, u'\} \in E'$ by at most 3 edges of length at most $\min\{d_{\max}, ||uu'||\}$ and adding all edges $\{u, v\}$ with $u \in U$, $v \in V \setminus U$ and $||uv|| \leq d_0$.

Step 3 is possible because by definition there is only an edge between two nodes $u, u' \in U$ in G if there are two nodes $v, v' \in V$ with $||uv|| \leq d_0$, $||u'v'|| \leq d_0$ and $||vv'|| \leq d_{\max}$.

Using this construction, all spanner results in the previous section hold, apart from the planarity property. Here, one would have to take care that when connecting the nodes in the dominating set in step 3, planarity is maintained, which is possible.

3.4. Contention. In all spanner constructions above, all nodes have equal roles. This, however, can be bad in areas with a high node density because then a lot of nodes have to coordinate their transmissions so that eventually messages can get through. An alternative solution is to partition the nodes into two groups: normal cluster nodes and cluster leaders, which we will also call *passive* and *active* nodes in the following. All communication is scheduled by the cluster leaders, or active nodes, in a sense that they determine who is allowed to transmit a message at a certain time point. This significantly simplifies the contention problem if there are only a few cluster leaders within a transmission range.

Ideally, the active nodes should form a connected set so that they can handle all non-local communication, and for each passive node there should be an active node within transmission range so that also all messages from and to passive nodes can be forwarded by the active nodes. Finding such a set of nodes is also known as the *connected dominating set problem*. Various distributed protocols have already been presented to find such a set. See, for example, [2, 9, 13, 16, 17]. As long as our model above with the cost function t is applicable, these protocols actually yield a connected dominating set of constant density, i.e., every node has only a constant number of dominating set nodes within its transmission range. In addition, any of the spanner constructions can be applied to the active nodes to further reduce the number of potential communication links while keeping the network connected. In this way, the effort of coordinating the transmissions between the active nodes can be kept at a low constant, no matter how many nodes there are in the network, so that the effort of scheduling message transmissions can scale to any number of nodes.

3.5. Mobility. As long as the nodes do not move around (and no node fails), the overlay network only has to be constructed only once. However, if the nodes move around, then frequent updates to the overlay network may be necessary. To limit the amount of updates, we need a mechanism that only connects nodes that can remain connected for a certain time. Certainly, this requirement can only be satisfied for two nodes v and w if the relative speed of v to w is small. This easiest way to take this into account is to add additional dimensions for the speed (see also [27]). Since speed in a 2-dimensional Euclidean space is a 2-dimension vector, this would result in a 4-dimensional space. Next we investigate whether our spanner constructions can still be used in this space.

Proximity graphs. For these graphs, only the distance between two nodes is relevant, not the dimension of the space. Hence, all the properties of proximity graphs are preserved.

Sector-based graphs. For Yao-graphs, the following result is known.

THEOREM 3.3 ([22, 25]). *Let V be any set of n points in $\mathbb{R}^d$ and let $0 < \theta < \pi/3$. Then the graph $YG_\theta(V)$ is a geometric spanner for V with stretch factor $\frac{1}{1-2\sin(\theta/2)}$. The number of cones needed to obtain an angle of θ is $O(d^{-1/2}(\frac{d^{3/2}}{\theta})^{d-1})$.*

Hence, the stretch factor is the same as for the 2-dimensional case, but the number of cones to obtain a certain angle θ grows exponentially in the dimension d.

Planar graphs. Certainly, planarity is not relevant any more in a more than 2-dimensional space, but the other spanner results about the various Delaunay graphs can be shown to hold as before.

4. Conclusions. In this paper we gave an overview of spanner constructions relevant for wireless ad hoc networks. We studied the performance of these constructions both in an idealized model and under more realistic assumptions. Interesting open problems in the future are how to route efficiently in these networks when using realistic communication and mobility models. For the UDG model and static nodes, there is already a large body of work on routing protocols (see, e.g., [5, 15, 18, 19]), but most of these results heavily rely on the assumption that the given overlay network is planar. Planarity, however, is hard to achieve under more realistic models, as we saw above. So further research is necessary.

REFERENCES

[1] K. Alzoubi, X.-Y. Li, Y. Wang, P. Wan, and O. Frieder, *Geometric spanners for wireless ad hoc networks*, IEEE Transactions on Parallel and Distributed Systems, 14 (2003), pp. 408–421.

[2] K. Alzoubi, P.-J. Wan, and O. Frieder, *New Distributed Algorithm for Connected Dominating Set in Wireless Ad Hoc Networks*, in Proceedings of the Thirty-Fourth Annual Hawaii International Conference on System Science (HICSS-35), IEEE Computer Society Press, 2002.

[3] F. M. auf der Heide, C. Schindelhauer, K. Volbert, and M. Grünewald, *Energy, congestion and dilation in radio networks*, in Proc. of the 14th ACM Symp. on Parallel Algorithms and Architectures (SPAA), 2002, pp. 230–237.

[4] P. Bose, L. Devroye, W. Evans, and D. Kirkpatrick, *On the spanning ratio of Gabriel graphs and beta-skeletons*, in Proc. of Latin American Theoretical Informatics Conference (LATIN), 2002.

[5] P. Bose, P. Morin, I. Stojmenovic, and J. Urrutia, *Routing with guaranteed delivery in ad hoc wireless networks*, in Proc. of 3rd Int. Workshop on Discrete Algorithms and Methods for Mobility (Dial-M), 1999, pp. 48–55.

[6] K. Clarkson, *Approximation algorithms for shortest path motion planning*, in Proc. of the 19th SIGACT Symposium, 1987.

[7] D. Dobkin, S. Friedman, and K. Supowit, *Delaunay graphs are almost as good as complete graphs*, Discrete Computational Geometry, (1990), pp. 399–407.

[8] D. Dubhashi, O. Häggström, A. Panconesi, and M. Sozio, *Irrigating ad hoc networks in constant time*, in Proc. of the 17th ACM Symp. on Parallel Algorithms and Architectures (SPAA), 2005, pp. 106–115.

[9] D. Dubhashi, A. Mei, A. Panconesi, J. Radhakrishnan, and A. Srinivasan, *Fast distributed algorithms for (weakly) connected dominating sets and linear-size skeletons*, in Proceedings of the fourteenth Annual ACM-SIAM Symposium on Discrete Algorithms (SODA-03), New York, Jan. 12–14 2003, ACM Press, pp. 717–724.

[10] D. Eppstein, *Handbook of Computational Geometry*, Elsevier, 2000, ch. Spanning trees and spanners, pp. 425–461.

[11] M. Grünewald, T. Lukovszki, C. Schindelhauer, and K. Volbert, *Distributed maintenance of resource efficient wireless network topologies*, in European Conference on Parallel Computing (EUROPAR), 2002, pp. 935–946.

[12] L. Jia, R. Rajaraman, and C. Scheideler, *On local algorithms for topology control and routing in ad hoc networks*, in Proc. of the 15th ACM Symp. on Parallel Algorithms and Architectures (SPAA), 2003, pp. 220–229.

[13] L. Jia, R. Rajaraman, and R. Suel, *An efficient distributed algorithm for constructing small dominating sets*, in PODC: 20th ACM SIGACT-SIGOPS Symposium on Principles of Distributed Computing, 2001.

[14] J. Keil and C. Gutwin, *Classes of graphs which approximate the complete Euclidean graph*, Discrete Computational Geometry, 7 (1992), pp. 13–28.

[15] E. Kranakis, H. Singh, and J. Urrutia, *Compass routing on geometric networks*, in Proc. of 11th Canadian Conference on Computational Geometry, 1999, pp. 51–54.

[16] F. Kuhn, T. Moscibroda, and R. Wattenhofer, *Radio network clustering from scratch*, in European Symposium on Algorithms (ESA), 2004.

[17] F. Kuhn and R. Wattenhofer, *Constant-time distributed dominating set approximation*, in Proceedings of the 22nd Annual ACM Symposium on Principles of Distributed Computing (PODC-03), New York, July 13–16 2003, ACM Press, pp. 25–32.

[18] F. Kuhn, R. Wattenhofer, Y. Zhang, and A. Zollinger, *Geometric ad-hoc routing: Of theory and practice*, in Proc. of the 22nd IEEE Symp. on Principles of Distributed Computing (PODC), 2003.

[19] F. Kuhn, R. Wattenhofer, and A. Zollinger, *Asymptotically optimal geometric mobile ad-hoc routing*, in Proc. of 6th Int. Workshop on Discrete Algorithms and Methods for Mobility (Dial-M), 2002, pp. 24–33.

[20] X.-Y. Li, G. Calinescu, P.-J. Wan, and Y. Wang, *Localized Delaunay triangulation with application in ad hoc wireless networks*, IEEE Transactions on Parallel and Distributed Systems, 14 (2003), pp. 1035–1047.

[21] X.-Y. Li, P.-J. Wan, and Y. Wang, *Power efficient and sparse spanner for wireless ad hoc networks*, in Proc. of IEEE International Conference on Computer Communications and Networks (ICCCN), 2001.

[22] T. Lukovszki, *New Results on Geometric Spanners and Their Applications*, PhD thesis, University of Paderborn, 1999.

[23] R. Rajaraman, *Topology control and routing in ad hoc networks: a survey*, SIGACT News, 33 (2002), pp. 60–73.

[24] S. Rao and W. Smith, *Approixmating geometrical graphs via spanners and banyans*, in Proc. of the 30th ACM Symp. on Theory of Computing (STOC), 1998, pp. 540–550.

[25] J. Ruppert and R. Seidel, *Approximating the d-dimensional complete euclidean graph*, in Proc. of the 3rd Canadian Conference on Computational Geometry (CCCG), 1991, pp. 207–210.

[26] C. Scheideler, *Performance Analysis of Mobile Ad Hoc Networks*, Nova Science Publishers, To appear in 2005, ch. Overlay Networks for Wireless Systems.

[27] C. Schindelhauer, T. Lukovszki, S. Rührup, and K. Volbert, *Worst case mobility in ad hoc networks*, in Proc. of the 13th ACM Symp. on Parallel

Algorithms and Architectures (SPAA), 2003, pp. 230–239.

[28] C. Schindelhauer, K. Volbert, and M. Ziegler, *Spanners, weak spanners, and power spanners for wireless networks*, in Proc. of 15th Annual International Symposium on Algorithms and Computation (ISAAC '04), 2004, pp. 805–821.

[29] Y. Wang and X.-Y. Li, *Localized construction of bounded degree and planar spanner for wireless ad hoc networks*, in DIALM-POMC '03: Proceedings of the 2003 joint workshop on Foundations of mobile computing, 2003, pp. 59–68.

[30] F. Xue and P. Kumar, *The number of neighbors needed for connectivity of wireless networks*, Wireless Networks, 10 (2004), pp. 169–181.

[31] A. C.-C. Yao, *On constructing minimum spanning trees in k-dimensional space and related problems*, SIAM Journal on Computing, 11 (1982), pp. 721–736.